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MODELAGEM DETERMINÍSTICA E ESTOCÁSTICA PARA PLANEJAMENTO DE OPERAÇÕES DE EMBARCAÇÕES SUPRIDORAS DE PLATAFORMAS EM LOGÍSTICA DE ÓLEO E GÁS OFFSHORE

Victor Anselmo Silva

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Orientadores: Virgílio José Martins Ferreira Filho
Juan Pablo Cajahuanca Luna

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To my beloved wife and beautiful daughters, for always supporting me, even when I was physically close to them, but mentally far.

Greetings

No words can describe suitably how grateful I am in the first place to God, for helping me to accomplish this endeavor called doctoral research. Whenever I look back on my initial day as a graduate student and realize everything that has happened since then, I see His hand. For He is good, all the time.

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MODELAGEM DETERMINÍSTICA E ESTOCÁSTICA PARA PLANEJAMENTO DE OPERAÇÕES DE EMBARCAÇÕES SUPRIDORAS DE PLATAFORMAS EM LOGÍSTICA DE ÓLEO E GÁS OFFSHORE

Victor Anselmo Silva

Fevereiro/2023

Orientadores: Virgílio José Martins Ferreira Filho
Juan Pablo Cajahuanca Luna

Programa: Engenharia de Produção

Esta tese propõe modelos de programação linear inteira-mista para solução do problema de planejamento de operações de embarcações conhecidas como PSVs (*platform supply vessels*), as quais têm como principal função transportar diversos tipos de suprimentos entre uma base de apoio offshore e unidades marítimas, tais como plataformas de produção e sondas de perfuração. Esse problema envolve decisões de nível tático e operacional. Três modelos são propostos. Um deles voltado para o problema tático de clusterização de plataformas, modelado como um problema de múltiplas mochilas com múltiplas dimensões, em que plataformas representam itens e, PSVs, mochilas. E outros dois modelos voltados para o problema operacional de roteirização e programação de PSVs, envolvendo múltiplos produtos, frota heterogênea com múltiplos compartimentos, restrições de programação, tais como janelas de tempo, possibilidade de planejar antecipadamente múltiplas viagens, e de realizar múltiplas visitas. Um desses modelos de roteirização considera incerteza sob a forma de atrasos, os quais interrompem temporariamente os serviços offshore, sendo tais atrasos oriundos de condições ambientais offshore adversas. Tal modelo é tratado como um problema de recurso de dois estágios. Para avaliar o desempenho desses modelos, foram geradas instâncias artificiais inspiradas em dados reais, contendo 87 plataformas, no caso do modelo de clusterização, e de 3 a 7 plataformas no caso dos modelos de roteirização, resultando em 5 a 20 pedidos de transporte para serem roteirizados. Um amplo conjunto de experimentos feito com tais modelos revela soluções de boa qualidade, obtidas em tempo hábil e com relevância econômica, tornando tais modelos adequados para uso prático.

Abstract of Thesis presented to COPPE/UFRJ as a partial fulfillment of the requirements for the degree of Doctor of Science (D.Sc.)

DETERMINISTIC AND STOCHASTIC MODELING FOR PLANNING OF PLATFORM SUPPLY VESSELS OPERATIONS IN OFFSHORE OIL AND GAS LOGISTICS

Victor Anselmo Silva

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This dissertation proposes mixed-integer linear programming models to solve the problem of planning operations of ships known as PSVs (*platform supply vessels*), which have as main function the transport of several types of supplies between an offshore supply base and maritime platforms, such as production units and drilling rigs. This problem involves decisions in tactical and operational levels. Three models are proposed. One of them focused on the tactical problem of platforms' clustering, modeled as an m-dimensional multiple knapsack problem, in which platforms represent items and, PSVs, knapsacks. And other two models focused on the operational problem of routing and scheduling of PSVs, including multiple commodities, heterogeneous and mutiple-compartment fleet, scheduling constraints, such as time windows, possibility of planning in advance multiple trips, and of performing multiple visits. One of these routing models considers uncertainty regarded as delays, which temporarily interrupt offshore services, and originate from adverse environmental offshore conditions. Such a model is treated as a two-stage recourse problem. To assess the performance of these models, artificial instances inspired from real data were designed, containing 87 platforms, in the case of the clustering model, and 3 to 7 platforms, in the case of the routing models, resulting in 5 to 20 transport orders to be routed. A broad set of experiments conducted with such instances reveals good quality solutions, achieved in acceptable time, and with demonstrated economical relevance, which turns these models suitable for practical use.

Sumário

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Lista de Abreviaturas

DEP	Deterministic Equivalent Problem, p. 56
E&P	Exploration and Production of Oil and Gas, p. 1
MILP	Mixed-integer Linear Programming, p. 5
MPCP	Maritime Platforms Clustering Problem, p. 6
PSVOPP	Platform Supply Vessel Operations Planning Problem, p. 3
PSV	Platform Supply Vessels, p. 2
RCI	Rounded Capacity Inequalities, p. 6
SAA	Sample Average Approximation, p. 6
SPR	Stochastic Program with Recourse, p. 6
VRP	Vehicle Routing Problem, p. 5
VSS	Value of the Stochastic Solution, p. 6, 65
d-PSVRSP	Deterministic Platform Supply Vessel Routing and Scheduling Problem, p. 6
s-PSVRSP	Stochastic Platform Supply Vessel Routing and Scheduling Problem, p. 6

Capítulo 1

Introduction

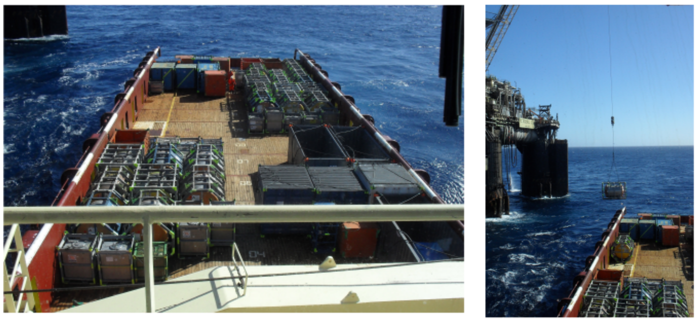
In 2021, the offshore production of petroleum and natural gas in Brazil corresponded, respectively, to 97 and 83% of the totals exploited by the country regarding those hydrocarbon sources. The Pre-salt reservoirs alone contributed with 74% of the petroleum produced and with 67.5% of the gas. Such numbers position Brazil in the 8^{th} and 30^{th} places in the world ranking of petroleum and natural gas producers, respectively, as seen in ANP (2022), with the national largest operator being Petróleo Brasileiro S.A., better known by the portmanteau Petrobras.

As the majority of the exploration and production of oil and gas (E&P) in Brazil is in the sea, the technological challenges to operate fields appear not only in traditional areas such as reservoir management, well construction and production engineering, but also in the transportation of personnel and cargoes between onshore facilities and maritime platforms.

In this dissertation, the term *cargoes* is overloaded in meaning to every type of supply that needs to be transported to maritime platforms, or from them to shore. Commonly, cargoes are handled either inside containers, in units, or even as bulk, and appear in three classes: deck cargo, comprised by all kinds of crane-handled material, which may appear as containerized items, chemical tanks, pipes, and special well equipment; liquid bulk, such as fuel oil (diesel), fresh water, drilling mud, and brine; and dry bulk, which includes cement, barite, and bentonite.

Depending on the type of platform in operation, on the service it may be performing, and on the current operational requirements, the cargo demand profile can change. Therefore, it is a key aspect for E&P business to achieve sound, cost-effective logistic plans for utilization of transport resources and onshore infrastructure. According to AAS *et al.* (2007), the area typically in charge of such plans is the so-called *offshore logistics*, also known as *upstream logistics*, which aims to provide the best possible service and operational continuity to offshore activities.

The key role in offshore logistics is played by the *platform supply vessels* (PSV), a ship type whose primary functions are logistic support and transportation of cargoes between an onshore infrastructure, commonly a supply base, and maritime platforms, such as rigs, production units and special service ships (BABICZ, 2015). A typical large PSV presents approximately 100 m of length, 18 m of beam, 900 m^2 of brute deck area, from which in general 70–80% are effectively used to accommodate deck cargoes, while the remainder serves to crew circulation. Figure 1.1 presents two views of a PSV in service. Figure 1.1a illustrates its deck space containing containers and chemical tanks, whereas Figure 1.1b shows a "lift"[1] operation.



(a) Deck area of a PSV. (b) Lifting operations.

Figura 1.1: PSV in service (LEITE, 2012).

Compartments for liquid and dry bulks can vary greatly from one vessel to another, but usual sizes for the largest vessels gravitate around 1500 m^3 for fuel oil, 2300 m^3 for fresh water, 3800 m^3 for drilling fluids and 400 m^3 for dry bulk (TIDEWATER, 2023). When in service offshore, a PSV can carry concomitantly, for instance, containers, diesel, and cement to be delivered for one or more platforms. Moreover, it can also bear deck cargoes that should be returned to shore.

PSVs are costly shuttle resources, usually chartered on an year-basis at daily rates near to USD [2]30,000.00 (MENDES VIANNA, 2016) for a large vessel, despite it can fluctuate considerably as a function of crude oil prices. PSVs can also be chartered to perform only a few offshore services, however the daily rates in such a condition tend to be even higher. If one includes fuel consumption at 9.3 tons/day, as reported by ADLAND *et al.* (2019), and fuel oil priced at USD 1,100.00/ton (OILMONSTER, 2022), the daily cost to maintain a vessel surpasses USD 40,000.00, excluding associated overhead costs. Since extensive offshore E&P operations employ dozens of such vessels, the logistics costs associated with a fleet of PSVs can easily be in the hundreds of millions, on an yearly basis.

In light of the context so far presented, this dissertation introduces the *platform supply vessel operations planning problem* (PSVOPP), whose general concern resides

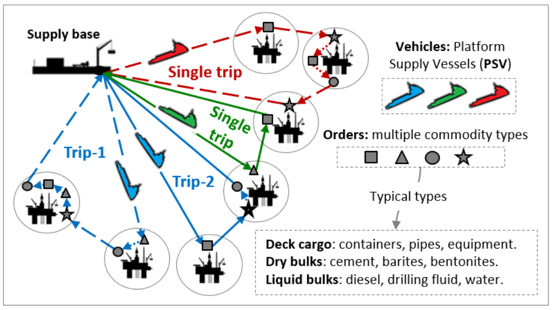
[1]Crane movement to deliver/receive an item (e.g., a container).
[2]United States Dollar.

in designing a tactic-operational plan to realize the transport of cargoes between an onshore facility and maritime platforms, at a cost-effective approach.

1.1 General dissertation's problem description

The PSVOPP consists of finding an optimal transport plan that yields: (i) a set of clusters, i.e., groups of platforms that demand cargo transportation; (ii) a set of routes for PSVs that can attend such a transport demand; and (iii) a time scheduling for such PSVs; aiming to minimize vessels usage, as well as costs related to fuel consumption. This objective needs to encompass tactical constraints, related to the clustering problem – such as maximum and minimum number of platforms per cluster, maximum supply base loading time per cluster – as well as operational ones, associated with the routing problem – including to meet the transport demand, respecting each vessel's capacity, and executing the offshore services as much as possible under time-related requirements. Figure 1.2 illustrates the offshore application context for the problem treated in this dissertation.

(a) Offshore E&P in Brazil LEITE (2012).



(b) Offshore operations example.

Figura 1.2: Application context example.

Given the aspects mentioned, the PSVOPP instantiates as follows. In large offshore oil and gas operations, it is usual to (re)organize platforms in clusters on a basis that mainly depends on rigs' location changes, variations of cargoes demand profile, and possibly on specific business needs, like maximum intra-cluster distance or maximum supply base loading time per cluster. Besides, operating with previously defined clusters provides predictability with respect to the supply base operations, due to the anticipated allocation of a PSV per cluster and the associated supply base time slot to operate cargoes ordered by the platforms of such a cluster. In turn, these scheme allows the onshore supply chain backwards the base, such as operator's in-house tasks, contractors' preparations, and the associated onshore transportation, to target specific vessel departure times, since each departure is oriented for a specific cluster.

The problem in this process of (re)organizing clusters regularly is the lack of specialized methods to assure good quality solutions, conversely to the widely adopted practice of developing approaches based on personalized heuristics and spreadsheets. Regardless of how clusters are designed, the execution of transport activities by PSVs is intended to fulfill, on time, orders of deck cargo, liquid, and dry bulks placed recurrently by platforms on a daily basis. In turn, this recurrent process allows one to regard the PSV activities as a periodic one, in the sense that platforms might be visited at a fixed frequency, e.g., twice a week. As an example, a PSV might visit a platform to deliver a large set of drilling pipes, whereas, in a second moment, it could collect waste containers, and deliver chemical tanks and diesel. However, fixing the number of visits *a priori*, as it is usually done in periodic schemes, can restrict the flexibility of the transport solutions designed, leading the extra PSV trips to accomplish orders that can not wait for certain visit day.

When a PSV is in service, pickup and delivery orders of deck cargo can concur for its deck space, which means that this vessel can concomitantly possess delivery and pickup orders of several platforms on its deck. With respect to tanks (or silos), there is usually no mixing of liquid (or dry) bulks, unless it is expressly allowed. During a route, there is no obligation for a vessel to execute simultaneous pickup and delivery, nor to fulfill all orders placed by a platform in a single visit. Moreover, it is possible that a different vessel attends part of that platform's demand. As an example, if a rig places three delivery orders, e.g., 100 m^2 of deck cargo, 500 m^3 of diesel and 300 m^3 of water, this demand could be split by type of commodity and attended separately, with a single vessel in either a single visit or multiple ones, or even with a second vessel, as long as no service time overlapping occurs. As offshore platforms have very limited deck space, it can be desirable to enforce that all pickups of deck cargo for certain platform be fulfilled before the deliveries of this cargo sort for that platform. This precedence does not necessarily require that pickups be collected immediately before deliveries. Also, establishing that rule does not mean a PSV must firstly collects all returning deck cargoes in its route, and then perform the deliveries.

Offshore logistics is an activity strongly dependent on service scheduling. For instance, an order can have a deadline. As an example, pickup orders include expensive rented equipment belonging to contractors, which can result in deadlines to have them delivered back to shore at the supply base. Beyond deadlines, a platform can also stipulate time windows inside which the services related to the orders it demands are preferably expected to occur. When a PSV arrives at the vicinity of a platform, it develops a different speed to safely get closer to that installation. Part of the PSV fleet that is not in service is available for scheduling and awaits at the supply base's vicinity, from which it can be scheduled in advance for one or more trips. It is assumed that there is no queue to access the supply base. Each of the remainder PSVs in the fleet has an estimated time of readiness, which comprises all necessary activities a vessel must accomplish before becoming available for use, such as navigation back to the supply base and cargo unloading, occasional compartment cleaning, and authority inspections. The time a PSV spends at the berth – i.e., the mooring area in the supply base where vessels operate cargoes – results from the amount of cargo being handled.

It is not rare that platforms can not be serviced by a vessel for some time interval, especially when other conflicting operations or special circumstances take place, such as a downtime of some cargo handling equipment (e.g., crane maintenance) or environmental conditions regarded unsafe to operate (e.g., prohibitive wind, waves or sea currents). Moreover, as long trips increase the transport plan's exposure to uncertainty, like adverse weather, it is salutary to limit how long a vessel stays offshore, with the objective to provide predictability on the moment each vessel will return to the base and be ready for a subsequent trip. Another concern of people in charge of executing the daily logistics operations is performing the offshore services in the shortest time possible, so that vessels' availability for upcoming trips is augmented. In this sense, a natural aspect to be optimized is the cumulative utilization time of the fleet. Besides, since the fleet is usually chartered for longer periods, such as a few years, there are no fixed operational costs for using a vessel, but only variable expenses given by fuel consumption in navigation or service. Conjugating efficiently all these issues to achieve a good quality maritime transport plan in offshore logistics is a problem hard to be solved, demanding specialized approaches such as mathematical optimization models, conversely to person-dependent heuristics and/or spreadsheets usually employed.

1.2 Objective and expected contributions

The main objective of this dissertation is to develop *mixed-integer linear programming* (MILP) optimization models to solve the *clustering* and the *vehicle routing problem* (VRP) existing in the PSVOPP. Regarding the specific dissertation contributions, they can be summarized as follows.

- An MILP model for the clustering problem, named *maritime platforms clustering problem* (MPCP), designed as an *m-dimensional multiple knapsack problem*, in which PSVs assume the role of multiple, heterogeneous knapsacks regarded as multiple-compartment resources, while maritime platforms interpret m-dimensional items, in which each dimension reflects a commodity type (e.g., deck cargo, diesel, water) to be carried in a specific, finite-capacity PSV's compartment.

- Two maritime routing and scheduling MILP models for cargoes transport activities performed by supply vessels. One of them is a rich-feature model, named *deterministic platform supply vessel routing and scheduling problem* (*d*-PSVRSP), consisting of a set of network and scheduling constraints that capture several complex features present in the offshore logistics system studied. The second routing model, called *stochastic* PSVRSP (*s*-PSVRSP), encompasses less features, however includes uncertainty data in its mathematical modelling, being formulated as a *deterministic equivalent program* (DEP) that represents a two-stage *stochastic program with recourse* (SPR). The routing models developed encode objective functions that address two main concerns in PSVs operations: vessels' fuel costs and efficient fleet utilization.

- A solution method for the MILP model related to the d-PSVRSP that integrates *rounded capacity inequalities* (RCIs) adapted from LAPORTE and NOBERT (1983) and WANG *et al.* (2021) as strengthening cuts to speed up the solution process[3], taking into account the cargo carrying capacity per compartment of a vessel.

- A solution method for the MILP model related to the *s*-PSVRSP based on the *sample average approximation* (SAA) method, including an estimate of the optimality gap regarding the original two-stage stochastic program, based on the procedure found in VERWEIJ *et al.* (2003). A solution quality evaluation based on the computation of the *value of the stochastic solution* (VSS), which measures how beneficial is to include uncertainty in the model.

- A literature review on studies that handle planning problems regarded as similar to those treated in this dissertation. The review presents a classification scheme for the routing models proposed in this work in comparison to other studies in the area of routing, including significant features for the practical problem.

- A set of realistic benchmark instances inspired by real logistics data of an E&P operator and extensive computational studies on the performance of the optimization models developed over those instances. Particularly, these models are relevant for practical use, given the following results obtained:

  – The MILP model introduced to solve the MPCP succeeded in 37% of the instances, for which optimality was achieved within 250 seconds, on

[3]The RCIs conceptualization and mathematical formulation are indeed contributions, however, the algorithms employed to separate them are not, but only their use in speeding up the solution process of the deterministic routing model

average, whereas the remainder 63% of them presented good quality solutions with average gap value of 2.7%, within 1 h set as run-time limit.

- The MILP routing model proposed to solve the d-PSVRSP solved to optimality 71.2% of the instances within approximately 2.5 minutes, on average. Other 28.4% of the instances presented final gap value of 10.1%, on average, at 0.5 h set as run-time limit. Less than 0.4% of the benchmark instances had no solution after 30 minutes of run-time.

- The MILP model designed to solve the s-PSVRSP could solve to optimality 77.5% of the instances within approximately 450 seconds, on average. The remainder 22.5% of the instances presented final gap value of 8.7% at 1 hour set as run-time limit. Introducing uncertainty in model indicates savings in USD within the range 7,032.00 – 28,444.00 per route.

1.3 Dissertation's organization

This dissertation is organized as follows. Chapter 2 contains a literature review on studies pertinent to problems treated in this work. Chapter 3 defines the problem treated in terms of its deterministic and stochastic aspects, as well as presents the mathematical notation used in the MILP models' formulations. Chapters 4, 5, and 6 present the mathematical modeling, computational studies, and discussions on the practical applicability of the models developed to solve instances of the MPCP, d-PSVRSP, and s-PSVRSP, respectively. Chapter 7 concludes the present work and points out possibilities for future studies.

Capítulo 2

Literature review

This chapter presents a literature review on the research topics pertinent to this dissertation: clustering of maritime platforms and routing and scheduling of PSVs. This review does not intend to be exhaustive on these topics, but only to provide a view on what has been studied in these areas that relates to the PSVOPP.

2.1 Studies in the dissertation's area

The problem of maritime platforms clustering in this dissertation resembles the widely studied *knapsack problem*, which has as seminal books MARTELLO and TOTH (1990) and KELLERER *et al.* (2004). In a knapsack problem, given a set of items and a finite-capacity knapsack, one aims to maximize the profit obtained from selecting a subset of these items to be carried in that knapsack, without infringing its capacity. The problem of clustering of maritime platforms can be translated to a knapsack-like one by interpreting such platforms as multiple-dimension items (m-dimensional), and PSVs as heterogeneous, multiple-compartment knapsacks.

Platforms are regarded as multiple-dimension items due to the existing types of cargo they demand, e.g., deck cargo, diesel, and water, being each of this types a dimension. As there is a fleet of PSVs available, the problem can also be seen as a multiple-knapsack one. The “profit"in practice arises from using the smallest possible number of PSVs (knapsacks) to form (carry) all maritime platforms (items) given. There is a subtle difference with respect to fundamental types of knapsack problems: every platform must be designated for a PSV, whereas not necessarily every item must be carried in a knapsack. The literature on knapsack problems is vast, including variants with the so-called *special constraints*, such as conflicting items, items' setups, color-constrained items, and multi-compartment knapsacks. The reader is referred to CACCHIANI *et al.* (2022a) for a detailed review on single-knapsack problems, and to CACCHIANI *et al.* (2022b) for a review on more elaborated, intricate variants, including the m-dimensional knapsack problems and special constraints.

Other traditional approaches for clustering of clients as an *a priori* task before routing appear in the classic algorithm of FISHER and JAIKUMAR (1981) and in the petal algorithm described by RYAN *et al.* (1993).

In offshore logistics, to the best of the knowledge of this dissertation's author, there are two studies that approach clustering of maritime platforms as a tactical approach before routing decisions. The first one is in the work of LONGHI (2014), in which the author developed an integer programming model to minimize the total intra-cluster distance using instances with 60 platforms. Similarly to this dissertation's study, a PSV is used as a capacitated transport resource to form clusters of platforms ranging from 4 to 6 platforms each. The constraints presented in the model are traditional ones, being related to the assignment of every platform to a cluster, while respecting the vessel's capacity. The final number of clusters is predefined, and a single commodity is considered.

The second study appears in SOARES and LEITE (2014). The clustering approach in such a study does not aim to form final clusters for routing, but only to partition the set of approximately 60 maritime platforms into smaller subsets with at most 15 platforms, in a manner that increases the tractability of an MILP routing model subsequently applied at each cluster. The authors employs two heuristic methods in their cluster formation procedure: a sweep heuristics, described in GILLETT and MILLER (1974), and a capacitated-clustering heuristics, further detailed in KOSKOSIDIS and POWELL (1992).

Concerning the second topic of this dissertation, vehicle routing, it is a very active research area, with huge advances in the 2000s. The books of GOLDEN *et al.* (2008) and TOTH *et al.* (2014), as well as the paper of VIDAL *et al.* (2020) provide reviews of several VRP variants and solution methods. In a few words, a VRP consists of finding an optimal set of routes to be completed by a fleet of vehicles in charge of attending a group of clients. Among the numerous solution mechanisms that have been developed for routing problems, the successful heuristic approaches include *tabu search* GENDREAU *et al.* (1994) and *genetic algorithms* BAKER and AYECHEW (2003), whereas the most efficient exact algorithms are those based on the *branch-price-and-cut* method PESSOA *et al.* (2020).

The progress achieved in solving more fundamental VRP variants provides incentive for one to explore challenging real life applications, which impose additional complications, such as multiple-compartment vehicles, multiple commodities competing for one of these compartments, intricate sets of constraints and, moreover, uncertainty factors, such as weather conditions. In this sense, the so-called "rich VRPs"emerge as routing problems whose models and solution algorithms are claimed proper for realistic applications. On this growing area, comprehensive surveys and taxonomy can be found in CACERES-CRUZ *et al.* (2014) and LAHYANI *et al.*

(2015), respectively.

The solutions for the problem of designing daily operations of PSVs have focused on VRP-like approaches, as they provide a mathematical framework suitable to approach that problem and constitute a widely known research path in combinatorial optimization. Aligned to that tendency, the routing problems in this dissertation embrace real life aspects and resembles archetypal VRP types, such as the *heterogeneous fleet* VRP, the VRP with *time windows*, and the VRP *with pickups and deliveries*.

Likewise, they present similarities with the *mixed vehicle routing with pickups and deliveries*, given the requirement to check deck's capacity at each visited node, as pickups and deliveries of deck cargoes compete for deck space. However, classical mixed problems impose that a client has either a pickup or delivery order, which is not the case in the present application. Both routing models also remember the *split delivery* VRP – since orders of distinct commodity types belonging to the same platform can be delivered (or picked up) at different moments with the same vessel or not – and the *multiple trips* VRP, due to the option of planning in advance two or more routes of a PSV.

As a start point to bring some of these features to practice in the VRP theme in offshore logistics, the paper of FAGERHOLT (2000) is one of the first works that proposes an MILP routing model applied to PSVs fleet sizing, mix and scheduling for the Norwegian company Equinor. The algorithm firstly enumerates all possible routes, then allocates voyages to PSVs in a periodic scheme to minimize costs. The author's approach considers 150% of the average demand, besides a few qualitative assessments to increase the solutions' capability to deal with uncertainty. Years later, AAS *et al.* (2007) developed a single PSV, pickup and delivery MILP routing model for deck cargoes, again for Equinor operations, including not only vessel capacity constraints, but also platform's deck space availability. In GRIBKOVSKAIA *et al.* (2008), the same problem is tackled, but their work presents a construction heuristics and a tabu search algorithm to handle large instances.

One decade past from the paper of FAGERHOLT (2000), other E&P operator started to step into more rigorous optimization tools to improve its logistics planning. In IACHAN (2009), the evolution of Petrobras on the systematic application of operations research is described, and efforts pointing to PSVs' activities are reported. ALMEIDA (2009) presents a genetic algorithm to solve a pickup and delivery maritime routing problem. Among the real life features modeled, the heuristics is capable of handling multiple commodities, heterogeneous fleet, transshipment (orders from platform-to-platform), and time windows. LOPES (2011) proposes simplified version to solve the routing problem tackled by ALMEIDA. The author solves such a problem using a *record-to-record* travel metaheuristics. Both articles report satisfactory results in comparison with real operations of Petrobras at Campos basin.

In SHYSHOU *et al.* (2011), a *large neighborhood search* heuristics is developed to solve a periodic vehicle routing problem. The central idea of the algorithmic search is assigning feasible voyages to PSVs. HALVORSEN-WEARE *et al.* (2012a) propose a similar model, yet, they treat more rigorously the scheduling aspect of the problem related to trip duration and spread of PSVs' departure times from the supply base. The paper of UGLANE *et al.* (2012) introduces another MILP model for routing and scheduling of PSVs with refueling tankers regarded as offshore hub structures. The authors propose a Dantzig-Wolfe decomposition and a branch-and-price methodology to solve the problem. A similar application is found in ASTOURES *et al.* (2016), although no specialized solution algorithm was developed, just the application of a commercial solver.

In SOARES and LEITE (2014), a single-commodity heterogeneous fleet routing problem with time windows is proposed. The study's approach is based on a cluster first-route second strategy, using seeds and sweep heuristics initially, then an exact model is solved. All three publications use realistic data from Brazilian Campos Basin. SOPOT and GRIBKOVSKAIA (2014) present a pickup and delivery routing problem with multiple commodities to plan PSV activities. The problem is solved using a combination of *variable neighborhood search* and simulated annealing, allowing each client to be visited at most twice to fully meet its demand. The paper of ALBJERK *et al.* (2015) introduces another pickup and delivery problem solved exactly for small instances with *dynamic programming* and approximately for larger cases with variable neighborhood search. At Equinor's request, the authors also try to cope with delivery disruptions by formulating constraints that better encode system's recovery policies.

FERNÁNDEZ CUESTA *et al.* (2017) solves a *periodic vehicle routing problem* with emergency PSV trips launched on demand to deal with surplus demand. An *adaptive large neighborhood search* heuristic is used. In KISIALIOU *et al.* (2018b), another periodic routing and fleet composition model for Equinor is developed and solved with that method. In their algorithm, strategies for swapping PSVs among routes are designed, aiming to provide tighter bounds for the cost objective. The work of AMIRI *et al.* (2019) embraces routing and onshore infrastructure location. The problem is introduced as a two-echelon mixed-integer nonlinear programming model for the location-routing problem of supply vessels with time windows, and solved for small instances with Lagrangian decomposition.

CRUZ *et al.* (2019) also compose routing and infrastructure optimization decisions in a single model. The authors present a comprehensive periodic routing model that allows one to assess fleet composition and supply base scheduling. The solution strategy decomposes the problem in four sub-models, each of them providing a lower bound to the subsequent one. In that work, routes for small instances are constrained to eight platforms, whereas for larger instances, the solution process starts from clusters previously defined by a Brazilian E&P operator regarded as study case. The approach found in VIEIRA *et al.* (2021) unifies the features seen in KISIALIOU *et al.* (2018b) and CRUZ *et al.* (2019). It introduces a branch-and-cut algorithm that solves small instances using an MILP periodic routing model and an adaptive large neighborhood search is developed to handle larger instances.

Recently, two aspects have gained more attention in offshore logistics: real-time routing, presented in KOVALSKI and QASSIM (2022), and optimization of vessel speed, introduced in ULSRUD *et al.* (2022). The first study approaches the supply vessel routing problem with random service requests treated as a Markovian decision process to handle uncertainty regarding (non)prioritized service requests. The model proposed optimize route decisions only after the realization of random variables, hence it does not include any stochastic data in the MILP model. The second study presented an adaptive large neighborhood search heuristic to optimize routing decisions with respect to pickup and delivery orders under the influence of weather conditions modeled as time-dependent sailing parameters, which greatly affects vessels' speed and their fuel consumption.

Out of the offshore logistics scope, some applications are worth to mentioned. The work of RUSSELL and URBAN (2008) designed a tabu-search to solve a routing problem with soft time windows and uncertain travel times, in which the corrective actions are the extent at which a window is violated. MENDOZA *et al.* (2010, 2011) model vehicles as multi-compartmentalized ones, treat the demand as stochastic, and use detour-to-depot as a recourse decision. The articles of LI *et al.* (2010) and TAŞ *et al.* (2014) propose another stochastic VRP with soft time windows and stochastic travel and service times. The second stage decisions include time window violation, but the authors also consider an extra recourse action in the form of vehicle driver's overtime.

CHRISTIANSEN *et al.* (2017) develop a multiple trip routing formulation based on a path-flow model with case-specific constraints to handle the problem of refueling customer ships from a given fleet of fuel supply vessels operating in the broader area of Pireaus Port, Athens, Greece. In XU *et al.* (2017), another rich routing and scheduling model is presented by the formulation of a multiple-visit routing problem with pickups and deliveries, which also considers multiple commodities. The application scenario is a major tobacco company in central China. The problem consists of minimizing transportation costs related to transferring of multiple raw material types among the company's plants. The authors propose an exact model and a local search heuristics to solve the problem.

SHI *et al.* (2018) present an application in home health care services that uses a routing model with simultaneous delivery and pickup of drugs and medical instruments. Travel and service times are the stochastic parameters adopted and the recourse actions designed are the unit's caregiver remuneration for the extra working time, total delay for all the patients in the route, and penalties for violating time windows. The study of GUTIERREZ *et al.* (2018) proposes a combination of a memetic algorithm with a greedy randomized adaptive search procedure (GRASP) to handle instances with uncertain pickup and delivery quantities regarded as arising from a Poisson distribution. The author use detour-to-depot as a recourse action, realized when the vehicle lacks the supply quantity required by the next client, or when such a vehicle does not have enough space to receive a pickup quantity.

In LI and LI (2020), the solution approach extends the tabu seach heuristic by introducing an adaptive aspect in the tabu list length. The authors' method improve the existing trade-off in longer (or shorter) tabu lists, since longer lists tend to provide better solutions, at greater computational efforts, whereas shorter ones may yield faster solution, yet locally optimal, possibly.

The problem of routing electric vehicles is tackled in KESKIN *et al.* (2021) employing an adaptive large neighborhood search heuristic, in which the stochastic parameter is the delay imposed to a vehicle at recharging stations. This delay may disturb the routing schedule to the point at which performing service at a client after the recharging process (wait in queue and charging itself) does not fit the time window offered by the client. As a recourse decision, such a client may be skipped and the associated costs of it considered in the problem's second stage.

At last, still in the routing theme, the study of ZAROUK *et al.* (2022) introduces a multi-objective heterogeneous vehicle routing and scheduling model with stochastic demand, soft time windows, and variable travel times, whose objectives include minimize fuel consumption and maximize customer satisfaction, the latter achieved by avoiding as much as possible the occurrence of services out of their targeted time windows. As second-stage recourse actions, the model considers a ratio of unsatisfied customer demand.

2.2 Studies in topics correlated to the dissertation's area

Beyond studies focusing on optimization-centered methods for logistics problems in the oil and gas industry, the literature demonstrates that stochastic simulation and combinations of that technique with optimization have also been chosen to better understand how uncertainty factors matter in decision making involving PSVs. For example, assessing the performance of a PSV fleet size and mix using stochastic simulation appears in MAISIUK and GRIBKOVSKAIA (2014), SILVA *et al.* (2017), and BASÍLIO (2017). The supply of diesel for maritime platforms' operational continuity is studied in DIUANA *et al.* (2016) and MOREIRA *et al.* (2019). The authors developed simulators to compare transportation policies for such a commodity. In SILVA *et al.* (2015), another case-specific discrete-event simulator is developed to detect bottlenecks in delivering/collecting of chemicals offshore. In these simulation studies mentioned, the routing scheme is usually an input, and the target is to find out good strategies to mitigate the effect of uncertainty on the daily operations.

There have also been attempts to conjugate optimization methods and simulation models, from which mixed procedures have been designed. Relevant examples are KISIALIOU *et al.* (2018a) and DE BITTENCOURT *et al.* (2021). In those works, the authors seek to provide optimized, yet to some extent uncertainty hedged, routing and scheduling plans for PSVs. Beside these studies, some authors propose planning models for PSV activities by introducing in their simulation frameworks a few optimizing actions and stochastic parameters to cope with unexpected changes offshore. Relevant and insightful works in this line are presented in KAISER (2010) for oil operations in Gulf of Mexico, and LEITE (2012) for the Brazilian Campos Basin operated by Petrobras.

2.3 Consolidated view

Concerning the topic clustering, the two studies related to the present work are LONGHI (2014) and SOARES and LEITE (2014). The first one is based on an exact MILP formulation, and so is the model for the MPCP. However, the model of LONGHI includes just traditional assignment constraints, handles a single commodity, and minimizes total distance. Whereas the model proposed in the present work considers additional business-pertinent constraints – such as a limitation of the maximum number of platforms per cluster which can not operate during night-shifts – deals with multiple types of commodities, and minimizes the number of clusters and the berth times, which constitute conflicting objectives.

The study of SOARES and LEITE uses heuristics to form clusters of platforms by searching for less costly solutions in terms of distance to be traveled. No special constraints, nor multiple commodities are considered, conversely to what is proposes in this work. The scarcity of studies related to methods for grouping maritime platforms suitably to business conditions is notable, and the present dissertation's approach collaborates to reduce the existing gap with an MILP model rich in real, practical problem aspects.

With respect to routing, Table 2.1 presents a classification scheme for the articles cited. The scheme is organized in eight classes: solution method, objective function (minimization), vehicle, commodity, service, route, stochastic parameter, and recourse action. Each of these classes subsumes relevant features of the problems tackled in those articles.

Tabela 2.1: Classification scheme for studies related to routing.

1 Solution method	**2 Min. obj. function**
1.1 Heuristic	2.1 Fixed cost
1.2 Exact	2.2 Variable cost
1.3 Hybrid	2.3 Utilization time
3 Vehicle	**4 Commodity**
3.1 Multiple vehicles	4.1 Multiple types
3.2 Heterogeneous fleet	4.2 Pickup and delivery
3.3 Multiple compartments	4.3 Transshipment
5 Service	**6 Route**
5.1 Time windows	6.1 Multiple visits
5.2 Service precedence	6.2 Multiple trips
5.3 Order deadline	6.3 Route duration limit
7 Stochastic parameter	**8 Recourse action**
7.1 Travel time	8.1 Violation (TW, overtime)
7.2 Service time	8.2 Detour-to-depot
7.3 Delay (or wait) time	8.3 Skip client (or demand)
7.4 Demand	8.4 Await

Table 2.2 shows a comparison of those articles, including the routing models introduced, d-PSVRSP and s-PSVRSP, regarding the classes/features introduced. In that table, the last column presents the total number of features detected per article considering features from the class vehicle up to recourse action. The last row indicates the occurrence of certain feature.

The 33 studies listed in Table 2.2 can be organized by the number of features they cover in 3 groups: low, 1 to 5 features; medium, 6 to 10; and high, for 11 or more features. This grouping reveals that 70% of the studies present a low number of features; 24% of them have a medium number of features; and only 6% of them include 11 or more real life aspects in their modeling. These numbers demonstrate that there still is a relevant gap in the literature concerning feature-rich routing applications. Particularly, the three VRP studies with the highest number of features are ALMEIDA (2009) and the two routing models proposed in this dissertation.

Nevertheless, it is possible to observe that even in studies with less features, such as AAS *et al.* (2007) and GRIBKOVSKAIA *et al.* (2008), relevant aspects for the offshore logistics business are contemplated, such as pickups and deliveries, and the possibility to perform multiple visits. On the other hand, rich-feature models like those in ALMEIDA (2009), CHRISTIANSEN *et al.* (2017), and the present ones cover broadly issues involved in routing and scheduling of PSVs.

Tabela 2.2: Works comparison. Deterministic VRPs models range from 1 to 21 and stochastic models from 22 to 33. “TF"stands for total features presented from classes 3 to 8.

ID	Article	1	1.1	1.2	1.3	2	2.1	2.2	2.3	3	3.1	3.2	3.3	4	4.1	4.2	4.3	5	5.1	5.2	5.3	6	6.1	6.2	6.3	7	7.1	7.2	7.3	7.4	8	8.1	8.2	8.3	8.4	*TF*
1	FAGERHOLT (2000)			X			X		X		X	X							X					X	X											5
2	AAS *et al.* (2007)			X					X							X				X			X													3
3	GRIBKOVSKAIA *et al.* (2008)		X						X							X				X			X													3
4	ALMEIDA (2009)		X				X	X			X	X	X		X	X	X		X		X		X		X											10
5	LOPES (2011)		X					X			X	X																								2
6	SHYSHOU *et al.* (2011)		X				X	X			X														X											2
7	HALVORSEN-WEARE *et al.* (2012a)			X			X		X		X	X							X					X	X											5
8	UGLANE *et al.* (2012)			X			X	X			X	X				X			X						X											5
9	SOPOT and GRIBKOVSKAIA (2014)		X				X	X								X							X													2
10	SOARES (2014)			X				X	X		X	X							X						X											4
11	ALBJERK *et al.* (2015)		X				X	X			X					X							X		X											4
12	ASTOURES *et al.* (2016)		X				X	X			X	X													X											3
13	FERNÁNDEZ CUESTA *et al.* (2017)		X				X	X			X					X									X											3
14	CHRISTIANSEN *et al.* (2017)			X			X	X			X	X	X		X				X		X			X												7
15	XU *et al.* (2017)				X			X			X				X	X							X													4
16	KISIALIOU *et al.* (2018b)		X				X	X			X	X							X						X											4
17	CRUZ *et al.* (2019)			X			X	X			X	X													X											3
18	AMIRI *et al.* (2019)			X			X	X			X	X							X		X		X		X											6
19	VIEIRA *et al.* (2021)				X		X	X			X	X							X						X											4
20	KOVALSKI and QASSIM (2022)			X				X												X			X													2
21	ULSRUD *et al.* (2022)		X				X	X			X	X				X					X		X													5
22	*d*-PSVRSP			X				X	X		X	X	X		X	X			X	X	X		X	X	X											11
23	RUSSELL and URBAN (2008)		X					X	X		X								X								X					X				4
24	MENDOZA *et al.* (2010)		X					X			X		X		X								X							X			X			6
25	LI *et al.* (2010)		X					X	X		X								X						X		X	X				X				6
26	MENDOZA *et al.* (2011)		X					X			X		X		X								X							X			X			6
27	TAŞ *et al.* (2014)			X				X			X								X								X	X				X				5
28	SHI *et al.* (2018)		X					X	X		X					X			X						X		X	X				X				7
29	GUTIERREZ *et al.* (2018)		X					X			X												X							X			X			4
30	LI and LI (2020)		X					X			X								X										X			X				4
31	KESKIN *et al.* (2021)		X				X	X			X								X						X				X					X		5
32	ZAROUK *et al.* (2022)		X					X			X	X				X			X						X					X				X		7
33	*s*-PSVRSP			X				X			X	X	X		X	X			X				X						X			X			X	10
–	Total		19	12	2		16	30	9		29	16	6		7	13	1		18	4	5		14	4	18		4	3	3	4		6	3	2	1	–

Concerning the solution method, 58% of the studies employ heuristic approaches; 36% of them use exact algorithms; and 6% based their solution approach in a hybrid strategy. Since VRPs are *NP-hard* optimization problems and real life cases often demand rapid, good-quality routing solutions, it is natural that have most of the solution mechanisms be heuristic-based.

The most frequent objective function term is the variable routing cost, appearing in 91% if the studies. This number agrees with the fact that routing applications usually take place on a operational level, in which costs are a function of the distance traversed and/or fuel consumed, for example. In 48% of the studies, minimizing variable expenses appears combined with fixed costs for using a vehicle (or vessel). An example of a situation that incur fixed costs is chartering an extra PSV temporarily, e.g., just for a few trips, a decision typically made in markets that allow spot chartering of vessels. Less frequently, variable costs also appear with some modeling aimed to reduce vehicle utilization, such as minimizing final route time, which occurred in 27% of the papers cited.

With respect to the vehicle, 12% of the studies developed single-vehicle routing model (AAS *et al.*, 2007; GRIBKOVSKAIA *et al.*, 2008; KOVALSKI and QASSIM, 2022; SOPOT and GRIBKOVSKAIA, 2014), since there is no mark for them regarding the feature *multiple vehicles* in the *vehicle* class. The other 88% of the papers considered a fleet of vehicles is available for use. Roughly 50% of the studies elaborate their models considering an heterogeneous fleet, whereas 18% of them cover the multi-compartment aspect.

Only 21% of the articles treat their routing problems as multiple commodity ones, despite oftentimes in real life the problems have more than one commodity type, specially in offshore logistics. However, there has been a concern towards including not only delivery orders, but pickups too, as can be seen in 39% of the studies cited. In offshore logistics, deck cargoes are continuously demanded, on a daily basis, having orders placed more frequently and steadily than diesel, water, and drilling bulks. Yet, the demand of such orders are not negligible in volume, usually takes long pumping times to a platform, and greatly affects the service time scheduling component of the problem. The genetic algorithm of ALMEIDA (2009) is a rather comprehensive study that includes various commodity types, pickups/deliveries, and still transshipment orders (platform-to-platform).

Regarding the service class of features, Table 2.2 shows that time windows are the most relevant aspect in planning a service, as 54% of the studies include this feature in their models. Indeed, whether offshore or onshore, time windows provide valuable planning information to logistic operations, as well predictability to clients. Following time windows by far come service precedence and deadline with roughly 15% of occurrence each.

On the route specifics, 54% of the studies limit the route duration, 42% of them provide models able to cope with multiple visits per client, and only 12% allow planning in advance for more than one trip[1]. All of these features are relevant for practical purposes, as they yield fine-grained planning, which in turn collaborate to better fulfilling the requested offshore services.

Beyond the features so far seen, Table 2.2 also shows what has been studied in terms of uncertainly modeling and associated recourse actions. The stochastic parameters listed in Table 2.1 appear practically uniformly among the studies that include random data in their models, which correspond to 11. Each of these parameters is present in nearly 32% of the cases. Besides, three studies consider both travel and service times as random parameters in their models. On the other hand, assuming violation as a recourse action is the most frequent option, responding for 55% of the studies, followed by detour-to-depot with 27%, skip client with 18%, and awaiting with just one study that also considers violation as a corrective action.

[1]Assuming a route is made of one or more vehicle trips, and a trip made of: departure from a supply base, offshore services' realization, and return to that base

Capítulo 3

Problem definition

This chapter defines the PSVOPP in terms of its decisions related to clustering of maritime platforms, as well as routing and scheduling of PSVs. In this dissertation, the clustering problem in the PSVOPP is referred to as *maritime platforms clustering problem* (MPCP), whereas its routing facet is separated in other two problems, the *deterministic platform supply vessel routing and scheduling problem* (d-PSVRSP), and a variant of it including uncertainty, named *stochastic* PSVRSP (s-PSVRSP).

The clustering and routing problems share some characteristics. As an example, the three problems involve knowing in advance the platforms' locations, PSV fleet capacity, and cargoes demand. The main aspects of each MILP routing model already appear in Table 2.2, however, a major difference in the s-PSVRSP compared to its deterministic variant is the introduction of uncertainty data in model. Given the relations existing in the problems that compose the PSVOPP, the sections 3.1 and 3.2 present a joint definition for them, separating aspects assumed as deterministic ones from those considered stochastic. Section 3.3 summarizes the notations introduced.

3.1 Deterministic aspects

Offshore platforms, belonging to a set denoted by $\mathcal{C}$, are organized into smaller subgroups called *clusters*, following some tactical-level requirements. As en example, each cluster should respect limitations on the minimum and maximum number of platforms, denoted by F and G, respectively, and on the maximum distance between any two platforms of it, denoted by E. Let $\mathcal{V}$ denote a set of PSVs. Together, the platforms of a cluster should approximately correspond, in amount of cargo demanded for delivery on a regular basis, to the capacity of a PSV $k \in \mathcal{V}$, denoted by $Q^k_q, q \in \mathcal{Q}$, in which $\mathcal{Q}$ is the set of cargo carrying compartments.

PSVs have varying sizes, which are expressed in *deadweight tonnage* (DWT) values. For instance, a PSV4500 is capable of carrying 4500 tons, including weights of cargo, fuel, fresh water, ballast water, provisions, passengers, and crew. Usually, the offshore supply service industry in E&P offers PSV1500, PSV3000, and PSV4500, which in the context of this dissertation are regarded as small (`S`), medium (`M`), and large (`L`) PSV sizes, respectively, with sizes $DWT_k \in \{1500, 3000, 4500\}, k \in \mathcal{V}$. The capacities adopted per commodity type for these PSV sizes are defined in the Appendix.

Clusters can then be designed by designating platforms to vessels available at different sizes, respecting each vessel's compartment capacity. Usually, only the largest vessels are considered, as they leverage the cargo transport capacity. However, smaller ones can also be used for less cargo-demanding clusters. Case a vessel $k \in \mathcal{V}$ is selected for a cluster design, a non-dimensional ratio expressed by

$$R^k = \frac{\max_{k \in \mathcal{V}} \{DWT^k\}}{DWT^k}, \tag{3.1}$$

is defined as a cost associated to that selection. Such a cost prioritizes the use of larger vessels.

Let $\mathcal{P}$ denote the set of commodity types demanded by the platforms of a cluster. Every type of commodity is compatible with exactly one vessel's compartment type, but different commodities may be allowed to share the same compartment space. Let $\mathcal{P}_q \subseteq \mathcal{P}$ denote the set of commodity types that are compatible with compartment type $q \in \mathcal{Q}$.

Per cluster, it is accepted at most H platforms with impossibility to operate with a PSV during night-shifts of onboard personnel. An indicator parameter, defined by $I_c \in \{0, 1\}, c \in \mathcal{C}$, informs what platform can ($I_c = 0$) or can not ($I_c = 1$) be serviced during the night. The platforms of a cluster request delivery orders whose average demand over a period of six months is denoted by $U_{cp}, c \in \mathcal{C}, p \in \mathcal{P}$. This demand should be handled at a supply base berth[1]. In turn, this requires a supply base berth time slot to operate the cargoes associated with such a demand. The berth limit slot to operate of a vessel is limited to BT hours per cluster.

At a berth, a vessel can operate commodities simultaneously, i.e., in parallel, for example: deck cargo being handled by crane lifts while, simultaneously, a hose is connected to that vessel to pump first water into it, than diesel afterwards. In other words, water and diesel are transferred serially, meanwhile crane lifts happen. Therefore, two additional commodity sets to represent this pair of ongoing operations for a vessel are defined: $\mathcal{P}_1, \mathcal{P}_2 \subset \mathcal{P}$. Performing such a parallelism is not an obligation, actually, it is very context dependent – e.g., cargo transportation urgency, availability of berth time slots, and operations safety matters – thus, it is a decision

[1]A berth is a supply base area that accommodates transport resources, such as ships, intended to develop cargo handling operations.

commonly made moments before a loading operation starts. However, for planning purposes of berth operations, parallelism is considered.

Let $\mathcal{O}$ denote the set of pickup and/or delivery orders requested by platforms in $\mathcal{C}$ on a daily basis. Let $\mathcal{P}_-, \mathcal{P}_+ \subseteq \mathcal{P}$ denote the sets of commodity types that are ready to be picked up and delivered, respectively. The (un)loading rate of commodity $p \in \mathcal{P}$ at the supply base is σ_p per unit.

Order $i \in \mathcal{O}$ represents either delivering to or picking up at platform $c^i \in \mathcal{C}$ some commodity $p^i \in \mathcal{P}$ in quantity $D_i \in \{L_{ip}, B_{ip}\}$, with its offshore service time being $ST_i = \sum_{p \in \mathcal{P}} \phi_p (L_{ip} + B_{ip})$, in which L_{ip} and B_{ip} are delivery and pickup quantities, respectively, ϕ_p is the efficiency for (un)loading cargoes that c^i develops to handle p. Let $\mathcal{O}_- := \{i \in \mathcal{O} : p^i \in \mathcal{P}_-\}$ denote the set of pickup orders. Associated with pickup order $i \in \mathcal{O}_-$ is a due time DT_i by which the task of unloading this order at the supply base has to be finished. For any platform $c \in \mathcal{C}$, offshore service overlapping is not allowed among its requested orders.

The service time for order i at platform c^i is expected to fall entirely within exactly one time window of the time windows set, denoted by $\mathcal{W}_i$. Time window $h \in \mathcal{W}_i$ is encapsulated by an earliest and an latest time point, ET_{ih} and LT_{ih}, respectively. Case the service start time of i violates ET_{ih}, the violation extent is charged hourly by at penalty ζ_i. Similarly, case the service finish time of i violates LT_{ih}, a penalty $\beta_i = 2\zeta_i$ is charged hourly for the violation extent. Violating LT_{ih} causes greater impact in the offshore operations than ET_{ih}, as finishing a service later than LT_{ih} damages the predictability of services ahead in the route, so the penalty value doubles. Waiting is allowed before a vessel starts to serve some $c^i \in \mathcal{C}, i \in \mathcal{O}$.

A PSV $k \in \mathcal{V}$ becomes available for loading at the supply base only at moment AT^k. If it is not used at that moment, it is assumed it awaits for use at the onshore base's vicinity. When vessel $k \in \mathcal{V}$ travels the distance $D_{c^i c^j}, i, j \in \mathcal{C}$ from c^i to c^j, given $c^i, c^j \in \mathcal{C}$, $i, j \in \mathcal{O} \cup \{0\}, i \neq j$, it takes setup time SE^k to leave c^i, navigation time N_{ij}^k from i to j, and safe positioning time SP^k before start to operate at c^j. For convenience, it is utilized the index 0 to represent the onshore base and $c^0 := 0$. Therefore, the total travel time from c^i to c^j is given by:

$$T_{ij}^k = \begin{cases} SE^k + N_{ij}^k + SP^k, & c^i \neq c^j, i, j \in \mathcal{O} \cup \{0\}, i \neq j \\ 0, & c^i \equiv c^j, i, j \in \mathcal{O} \cup \{0\}, i \neq j \end{cases} \tag{3.2}$$

A *trip* is a sequence of the following tasks associated with a vessel: (i) loading onto that vessel at the onshore base the commodities that are ready to be delivered; (ii) serving orders offshore (including delivery and/or pickup orders); (iii) unloading onto the onshore base the pickup commodities collected along this trip. Vessel $k \in \mathcal{V}$ is allowed to perform at most L^k consecutive trips, while every trip's duration should not exceed TD^k. A *route* is a set of one or more consecutive trips performed by a vessel.

Assuming a fleet of PSVs is already chartered for a few years, there will be no operational costs related to vessels' daily rates. It is also assumed that there is no short-term PSVs' chartering for one to accomplish a few offshore services. With that given, the quantities of interest for the routing problem resides in the operational level, being specifically the fuel expenditures and vessel utilization time. The fuel expenditure is defined as follows. Every vessel $k \in \mathcal{V}$ available at AT^k, which has not been scheduled to accomplish a route, awaits at the base's vicinity consuming fuel at hourly price θ^k. Case k is scheduled, it consumes fuel at hourly prices denoted by: φ^k, which is the fuel cost per hour when k is operating at the supply base; γ^k, which is the fuel cost per hour when k is navigating; and δ^k, which is the fuel cost per hour when k is enrolled in offshore, service-related tasks, such as cargo handling, waiting times, or setups. Vessel times related to setup when leaving a platform, safe positioning when arriving at it, as well as hourly costs related related to fuel consumption for each vessel in its various modes (navigation, service, waiting and setups), are all defined in the Appendix.

The vessel utilization time comprehends the time interval starting at AT^k, for vessel $k \in \mathcal{V}$, and finishing by the end of such a vessel's route [2]. Let ξ^k be a non-dimensional, positive factor devised for each $k \in \mathcal{V}$ to promote the use of smaller vessels whenever possible at each routing opportunity, implicitly leaving more vessel capacity for future use in the daily logistic routine. This parameter is defined in the Appendix, specifically in 7.1.

The goal of MPCP is to decide for a feasible set of maritime platform clusters, such that the number of clusters and berth times are minimized, as well as tactical requirements are respected. Regarding the routing problems, the goal of the *d*-PSVRSP is to decide for a feasible set of routes and schedules for PSVs such that fuel costs and vessel utilization times, are minimized as a composite function α-weighed, while all operational requirements are met, such as orders fulfilled, and constraints respected, including vessels' capacity and time-related requirements.

3.2 Stochastic aspects

The transport of cargoes in offshore logistics is subjected to uncertainties. Therefore, the transport plans can be perturbed or even flaw. There are two classes of uncertainty that logistics planners and analysts in offshore logistics attempt to cope with: *endogenous* and *exogenous*.

[2]The route of vessel k finishes when the last pickup of the last trip performed is unloaded onto the base.

Endogenous uncertainty manifests in several ways, such as: (i) a PSV fault, what temporarily removes the resource from operations to be repaired (corrective maintenance); (ii) a transitory unavailability of a platform to receive its orders due to a shortage of platform's deck space, in turn caused by an excess of pickups on board; (iii) a conflict created by the impossibility for a PSV to serve a platform, given an there already exists another operation ongoing, like personnel diving for inspection, causing the vessel to wait in queue; and (iv) fluctuations on the efficiency to transfer cargo to the platform, or from it to the vessel.

Exogenous uncertainty is preponderantly originated from environmental conditions, which are obviously uncontrollable. The major factors influencing the vessel's performance offshore are wind, sea current and waves. Depending on their severity, isolated or combined, the transferring of cargoes between a PSV and a platform can be simply delayed to start, interrupted, or even canceled.

In this dissertation, only the exogenous uncertainty is considered, being regarded as delays that "push"the service start moment ahead in time, at a duration corresponding to the delay size. It is assumed that such delays arise from unfavorable offshore environmental conditions, which temporarily prohibits a vessel to operate, resulting in unplanned waiting times offshore. Other performance-related variations such as navigation velocity and cargo loading/unloading rates are assumed to be minor, therefore considered as deterministic. These delays are modeled as follows.

After certain vessel $k \in \mathcal{V}$ finishes the travelling from platform c^i to platform c^j, given $c^i, c^j \in \mathcal{C}, i, j \in \mathcal{O} \cup \{0\}, i \neq j$, it can encounter environmental conditions prohibitive to perform order's j service safely, which in turn disturbs the offshore agenda planned for that vessel, for platform c^j, and for subsequent platforms in the route. In this context, it is defined a set of scenarios Ω and a stochastic parameter $S_{iw} \geqslant 0, i \in \mathcal{O} \cup \{0\}, \omega \in \Omega$ to represent exogenous uncertainty that manifests as a delay originated from those unsafe operational conditions, which leads to waiting times before starting to serve order j. All delays a vessel can incur offshore also represent extra costs related to δ^k, the fuel consumption cost when k is offshore in service or waiting, planned or not.

Within the described context, the goal of the s-PSVRSP is to decide for a feasible set of routes and schedules for PSVs such that the total fuel cost is minimized, while all operational requirements are met, such as orders fulfilled, and constraints respected, including vessels' capacity and time-related requirements. In addition to that, such a goal must also provide mechanisms to mitigate, whenever possible, realizations of exogenous uncertainty, which impact the offshore logistic services negatively.

3.3 Consolidated notation

The notation presented in sections 3.1 and 3.2 is summarized as follows.

Sets

$\mathcal{C}$	Set of maritime platforms, defined as: $\mathcal{C} \coloneqq \{1, 2, \ldots, b\}$.
$\mathcal{P}$	Set of commodity types.
$\mathcal{P}_{-} \subseteq \mathcal{P}$	Set of commodity types that are ready to be picked.
$\mathcal{P}_{+} \subseteq \mathcal{P}$	Set of commodity types that are ready to be delivered.
$\mathcal{P}_1 \subset \mathcal{P}$	Subset 1 of commodities allowed to operate in parallel at a berth.
$\mathcal{P}_2 \subset \mathcal{P}$	Subset 2 of commodities allowed to operate in parallel at a berth.
$\mathcal{O}$	Set of orders requested by maritime platforms, defined as: $\mathcal{O} \coloneqq \{1, 2, \ldots, n\}$.
$\mathcal{O}_{-}$	Set of pickup orders, defined as: $\mathcal{O}_{-} \coloneqq \{i \in \mathcal{O} : p^i \in \mathcal{P}_{-}\}$.
$\mathcal{W}_i$	Set of time windows for order $i \in \mathcal{O}$ requested by platform $c^i \in \mathcal{C}$.
$\mathcal{V}$	Set of PSVs.
$\mathcal{Q}$	Set of compartment types for a vessel.
$\mathcal{P}_q \subseteq \mathcal{P}$	Set of commodity types compatible with compartment type $q \in \mathcal{Q}$.
Ω	Set of scenarios.

Parameters

F	Minimum number of platforms per cluster.
G	Maximum number of platforms per cluster.
H	Maximum number of platforms per cluster that can not operate during night-shifts.
E	Maximum distance between any two platforms of a cluster.
BT	Maximum supply base berth time per cluster.
I_c	An indicator parameter, defined by $I_c \in \{0, 1\}, c \in \mathcal{C}$, to inform what platform can ($I_c = 0$) or can not ($I_c = 1$) be serviced during night-shifts.
DWT^k	Deadweight tonnage of vessel, given by $DWT^k \in \{1500, 3000, 4500\}, k \in \mathcal{V}$.
R^k	Cost associated with selecting vessel $k \in \mathcal{V}$ to design a cluster, defined in 3.1.
ϕ_p	Loading/unloading rate of commodity $p \in \mathcal{P}$ at a platform.
σ_p	Loading/unloading rate of commodity $p \in \mathcal{P}$ at the supply base.
B_{ip}	Pickup quantity of commodity $p \in \mathcal{P}$, for order $i \in \mathcal{O}$.
L_{ip}	Delivery quantity of commodity $p \in \mathcal{P}$, for order $i \in \mathcal{O}$.
U_{cp}	Average demand of commodity $p \in \mathcal{P}$ for platform $c \in \mathcal{C}$.
D_i	Quantity of commodity $p^i \in \mathcal{P}$ for order $i \in \mathcal{O}$ to be either delivered to or picked up at platform $c^i \in \mathcal{C}$.
ST_i	Service time for order $i \in \mathcal{O}$.

ET_{ih}	Earliest time point for time window $h \in \mathcal{W}_i$ for order $i \in \mathcal{O}$.
LT_{ih}	Latest time point for time window $h \in \mathcal{W}_i$ for order $i \in \mathcal{O}$.
ζ_i	Penalty per hour for violating ET_{ih}, $i \in \mathcal{O}$, $h \in \mathcal{W}_i$.
β_i	Penalty per hour for violating LT_{ih}, $i \in \mathcal{O}$, $h \in \mathcal{W}_i$, defined as: $\beta_i = 2\zeta_i$.
DT_i	Due time for pickup order $i \in \mathcal{O}_-$, by which the task of unloading this order onto the onshore base has to be finished.
Q_q^k	Capacity of compartment type $q \in \mathcal{Q}$ for vessel $k \in \mathcal{V}$.
AT^k	Moment at which vessel $k \in \mathcal{V}$ becomes available for loading at the base.
N_{ij}^k	Navigation time for vessel $k \in \mathcal{V}$ from c^i to c^j, for $c^i, c^j \in \mathcal{C}$, $i, j \in \mathcal{O} \cup \{0\}$, $i \neq j$.
SE^k	Setup time for vessel $k \in \mathcal{V}$ before it departures from a platform.
SP^k	Safe positioning time for vessel $k \in \mathcal{V}$ when arriving at a platform.
T_{ij}^k	Total travel time for vessel $k \in \mathcal{V}$ from c^i to c^j, $c^i, c^j \in \mathcal{C}$, $i, j \in \mathcal{O} \cup \{0\}, i \neq j$, defined in 3.2.
L^k	Maximum number of consecutive trips allowed for vessel $k \in \mathcal{V}$.
TD^k	Maximum trip duration for vessel $k \in \mathcal{V}$.
θ^k	Fuel cost per hour for $k \in \mathcal{V}$ awaiting at the base's vicinity.
φ^k	Fuel cost per hour for $k \in \mathcal{V}$ operating at the supply base.
γ^k	Fuel cost per hour for $k \in \mathcal{V}$ in navigation.
δ^k	Fuel cost per hour for $k \in \mathcal{V}$ enrolled in service-related tasks offshore, such as cargo handling, waiting times, or setups.
ξ^k	Non-dimensional, positive factor devised for each $k \in \mathcal{V}$ to promote the use of smaller vessels.
α	Weight parameter, defined as: $\alpha \in [0, 1]$.
S_{iw}	Delay that a vessel incurs before serving order $i \in \mathcal{O} \cup \{0\}$ at scenario $\omega \in \Omega$.

Capítulo 4

Clustering of maritime platforms

This chapter contains the mathematical modeling of the MPCP. Besides, it also presents a series of experimental results obtained from solving artificial instances inspired from real life data of an oil and gas operator that develops its offshore logistics activities in southeast Brazilian waters, and a discussion on the practical application of the method proposed.

4.1 Mathematical modeling

This section presents an MILP formulation for the MPCP.

4.1.1 Specific notation

This section presents the notation necessary specifically for the MPCP.

Sets

$\mathcal{C}$	Set of maritime platforms, defined as: $\mathcal{C} := \{1, 2, \ldots, n\}$.
$\mathcal{D}$	Set of combinations of platform pairs, defined as: $\mathcal{D} := \{(c'c'') \in \mathcal{C} \times \mathcal{C} : c' \neq c'', c' < c''\}$.
$\mathcal{P}$	Set of commodity types.
$\mathcal{P}_1 \subset \mathcal{P}$	Subset 1 of commodities allowed to operate in parallel at a berth.
$\mathcal{P}_2 \subset \mathcal{P}$	Subset 2 of commodities allowed to operate in parallel at a berth.
$\mathcal{V}$	Set of PSVs.
$\mathcal{Q}$	Set of compartment types for a vessel.
$\mathcal{P}_q \subseteq \mathcal{P}$	Set of commodity types compatible with compartment type $q \in \mathcal{Q}$.

Parameters

F	Minimum number of platforms per cluster.
G	Maximum number of platforms per cluster.

H	Maximum number of platforms per cluster that can not operate during night-shifts.
E	Maximum distance between any two platforms of a cluster.
BT	Maximum supply base berth time per cluster.
I_c	An indicator parameter, defined by $I_c \in \{0, 1\}, c \in \mathcal{C}$, to inform what platform can ($I_c = 0$) or can not ($I_c = 1$) be serviced during night-shifts.
R^k	Cost associated with selecting vessel $k \in \mathcal{V}$ to design a cluster, defined in 3.1.
σ_p	Loading/unloading rate of commodity $p \in \mathcal{P}$ at the supply base.
U_{cp}	Average demand of commodity $p \in \mathcal{P}$ for platform $c \in \mathcal{C}$.
Q^k_q	Capacity of compartment type $q \in \mathcal{Q}$ for vessel $k \in \mathcal{V}$.
α	Weight parameter, defined as: $\alpha \in [0, 1]$.

4.1.2 Modeling

This section presents an MILP formulation for the MPCP. The constraints and objective function of this formulation are introduced in “blocks". The first block presents knapsack-like constraints to associate platforms to vessels. The second block defines constraints that impose limits on the number of platforms per cluster. The third block models constraints to limit the number of platforms, per cluster, which can not operate with PSVs during night shifts. The fourth block constrains the distance among the platforms of a cluster. The fifth block defines the loading time at the supply base that each cluster generates. The last block presents the objective function.

Knapsack constraints. Given a vessel $k \in \mathcal{V}$, which plays the role of a knapsack, let a binary variable y^k be 1 if and only if k is used, and a binary variable x^k_c be 1 if and only if maritime platform $c \in \mathcal{C}$, playing the role of an item, is designated to k. Constraints (4.1) enforce that every maritime platform must be designated for a vessel. Constraints (4.2) impose that the capacity of a vessel's compartment be respected regarding the commodity it is allowed to carry.

$$\sum_{k \in \mathcal{V}} x^k_c = 1 \qquad \forall c \in \mathcal{C} \tag{4.1}$$

$$\sum_{p \in \mathcal{P}_q} \sum_{c \in \mathcal{C}} U_{cp} x^k_c \leqslant Q^k_q y^k \qquad \forall k \in \mathcal{V}, \forall q \in \mathcal{Q} \tag{4.2}$$

Cluster's size limitation constraints. Constraints (4.3) impose a minimum number of platforms per cluster. This constraint is included in the model only if

$F > 1$. Constraints (4.4) impose a maximum number of platforms per cluster.

$$\sum_{c \in \mathcal{C}} x_c^k \geqslant F y^k \qquad \forall k \in \mathcal{V} \tag{4.3}$$

$$\sum_{c \in \mathcal{C}} x_c^k \leqslant G y^k \qquad \forall k \in \mathcal{V} \tag{4.4}$$

Night-shift limitation constraints. Constraints (4.5) impose a maximum number of platforms per cluster that are not allowed to have vessel operations at night.

$$\sum_{c \in \mathcal{C}} I_c x_c^k \leqslant H y^k \qquad \forall k \in \mathcal{V} \tag{4.5}$$

Intra-cluster distance limitation constraints. Let a binary variable $u_{\hat{c}\bar{c}}^k$ be 1 if and only if platforms $\hat{c}, \bar{c} \in \mathcal{C}$ are designated to vessel $k \in \mathcal{V}$. Together, constraints (4.6) and (4.7) impose a maximum distance between any two platforms of a cluster.

$$u_{\hat{c}\bar{c}}^k \geqslant x_{\hat{c}}^k + x_{\bar{c}}^k - 1 \qquad \forall k \in \mathcal{V}, \forall (\hat{c}, \bar{c}) \in \mathcal{D} \tag{4.6}$$

$$D_{\hat{c}\bar{c}} u_{\hat{c}\bar{c}}^k \leqslant E y^k \qquad \forall k \in \mathcal{V}, \forall (\hat{c}, \bar{c}) \in \mathcal{D} \tag{4.7}$$

Supply base loading time limitation constraints. Let a binary variable m^k be 1 if and only if the commodities loaded by vessel $k \in \mathcal{V}$ and associated with set $\mathcal{P}_2$ result in a berth time greater than that obtained from commodities in $\mathcal{P}_1$. Let a non-negative variable t^k denote the supply base loading time for vessel k. Let a non-negative variable d denote an upper bound for any supply base loading time. Constraints (4.8) impose an upper bound on any vessel's loading time. Constraints (4.9) impose a maximum loading time. Constraints (4.10)– (4.13) define the supply base loading time for a vessel k as follows. Constraints (4.10) and (4.11) define the cumulative berth time for k according to the commodities that appear in $\mathcal{P}_1$, whereas constraints (4.12) and (4.13) define the cumulative berth time for such a vessel according to the commodities that appear in $\mathcal{P}_2$. The berth time will result as the maximum time among the cumulative berth times produced from $\mathcal{P}_1$ and $\mathcal{P}_2$. If $m^k = 0$, the berth time t^k will be defined by constraints (4.10) and (4.11), i.e., it will arise solely from summation of the times related to handling the cargoes whose commodity types appear in $\mathcal{P}_1$. In turn, constraints (4.12) and (4.13) will be not binding. Case $m^k = 1$, the berth time t^k will be defined by constraints (4.12) and (4.13), i.e., it will arise solely from summation of the times related to handling the cargoes whose commodity types appear in $\mathcal{P}_2$. In turn, constraints (4.10) and (4.11) will be not binding. The big-M value M_1 is defined in the Appendix.

$$t^k \leqslant d \qquad \forall k \in \mathcal{V} \tag{4.8}$$

$$t^k \leqslant BTy^k \qquad \forall k \in \mathcal{V} \tag{4.9}$$

$$t^k \geqslant \sum_{c \in \mathcal{C}} \sum_{p \in \mathcal{P}_1} \sigma_p U_{cp} x_c^k \qquad \forall k \in \mathcal{V} \tag{4.10}$$

$$t^k \leqslant \sum_{c \in \mathcal{C}} \sum_{p \in \mathcal{P}_1} \sigma_p U_{cp} x_c^k + M_1 m^k \qquad \forall k \in \mathcal{V} \tag{4.11}$$

$$t^k \geqslant \sum_{c \in \mathcal{C}} \sum_{p \in \mathcal{P}_2} \sigma_p U_{cp} x_c^k \qquad \forall k \in \mathcal{V} \tag{4.12}$$

$$t^k \leqslant \sum_{c \in \mathcal{C}} \sum_{p \in \mathcal{P}_2} \sigma_p U_{cp} x_c^k + M_1(1 - m^k) \qquad \forall k \in \mathcal{V} \tag{4.13}$$

Objective function. The problem's objective is to minimize the number of clusters and the berth time bound from a composite function weighed by α. This objective is presented in (4.14).

$$\underset{y,d}{\text{Min}} \ \alpha \sum_{k \in \mathcal{V}} R^k y^k + (1 - \alpha) d \tag{4.14}$$

The goal of the MPCP is to perform the minimization expressed in (4.14), including decision variables subject to constraints (4.1) – (4.13).

4.2 Computational studies

In this section the instances generation process is described, as well as the computational results obtained from applying the method proposed to solve the MPCP. Implementations are made in Python 3.10.6 and all MILP models are solved using the Gurobi Optimizer 10.0.0 through the Python application programming interface, with all settings default, except the following: `MIPGap = 0.5%`, `TimeLimit = 3600` seconds, `NoRelHeurWork = 7`, `MIPFocus = 3`, `Cuts = 1`, and `Threads = 1`.

The gap value reported by the solver is defined as: $gap = \frac{UB - LB}{UB} \times 100$, in which UB stands for "upper bound"and LB stands for "lower bound". These parameter values were defined empirically from some tests made with a few instances, from which solutions were obtained faster than in the case that only the solver's default parameters were set. The `NoRelHeurWork` parameter intensifies the heuristic solver's search for solutions before the internal solver's solution process starts, which leads one to obtaining primal solutions of good quality. The `MIPFocus` parameter intensifies the solver's effort to improve the lower bound of the problem. The

Cuts parameter promotes a moderate generation of cutting planes to speed up the solution process. The Threads parameter limits the number of processing threads offered to the solver. Higher values for this parameter deliver more processing power for the solver to manage along its solution process. The reader is referred to the solver's website GUROBI (2023) for further details about these parameters.

All computations were performed on an Intel Xeon CPU W-10885M running at 2.40 GHz. A total of 32GB of available RAM was shared among 12 copies of the model running in parallel. Each instance was solved by one copy of the model using a single thread. All CPU times and relative optimality gap values presented as results in this section are calculated as arithmetic means.

4.2.1 Benchmark instances

A set of 87 platforms belonging to two oil and gas basins is considered for the instances' design. The location of these platforms are illustrated in Figure 4.1. There are 10 clusters identifiable as BCa, BCb, ..., ESc, which correspond to 35 platforms, whereas the cluster information for other 52 platforms was not available for studies at the time that data was collected. These platforms whose clusters are not defined appear in the figure with as a generic label "NAv", which stands for "not available". Some descriptive statistics about the 87-platform group are: the closest platform from the supply base is located at 93 km and the farthest at 213 km. Among the platforms, the minimum distance is nearly zero km (side-by-side platforms), the average is 41 km, the maximum is 134 km, and 90% of the platforms locate within 80 km from each other.

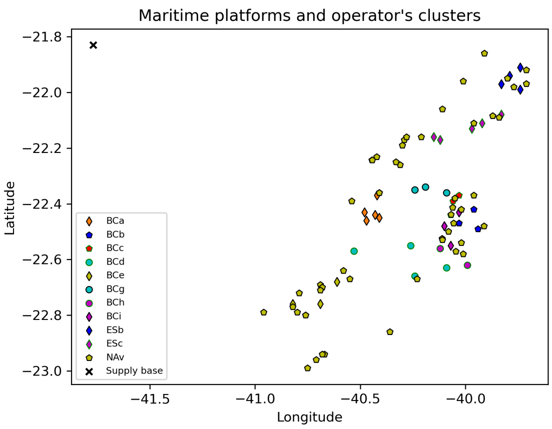
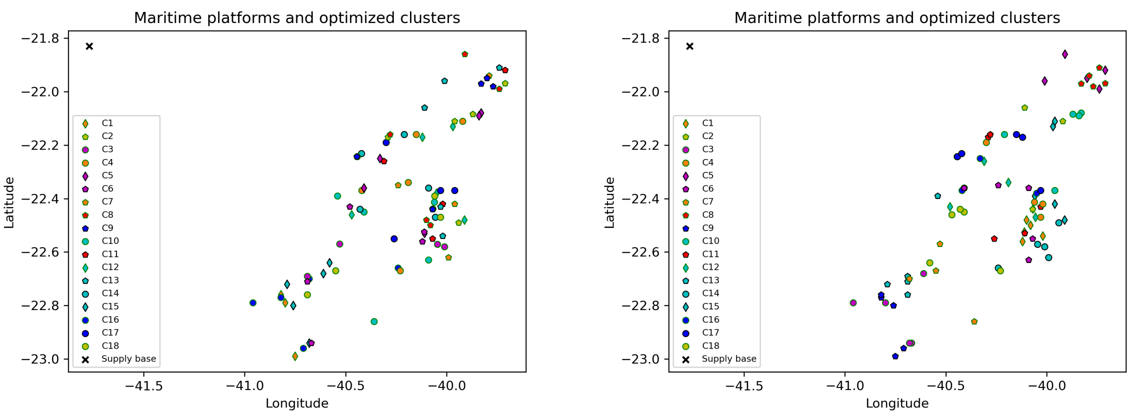


Figura 4.1: Platform clusters and supply base locations.

The instances were generated always with 87 platforms. The parameter values used to produce different instance versions were: $\alpha \in \{0, 0.25, 0.5, 0.75, 1\}$, $G \in \{4, 5\}$, and $D_{ij} \in \{80, 45\}, i, j \in \mathcal{C}$. The design of clusters use to consider a fixed demand for each commodity type. Yet, aiming to mimic some variability over time, which could trigger a clusters' reconfiguration, the demand of diesel and water was assumed to variate, since not all platforms demand diesel and/or water. Thus, it was adopted 3 combinations of demand values for diesel and water, in addition to the parameters α, G, and D_{ij} already selected to variate. The demand of deck cargo is assumed constant, and so are the values of the following parameters: $F = 3$, $H = 3$, and $BT = 15$ hours. In total, the number of feasible instances generated is $3_{\text{comb}} \times 5_{\alpha} \times 2_{G} \times 2_{D_{ij}} = 60$, in which "comb"stands for the diesel and water demand profile combinations, whereas the other subscript indexes refer to the parameter selected to vary. Regarding the vessels, also a total of 3 combinations of mixed-size PSVs fleet, i.e., `M` and `L`, containing 29 PSVs each fleet, were made available for every instance. Such combinations do not interfere in the total number of instances, as they are generated together with the 3 demand profiles of diesel and water.

4.2.2 Performance

This section demonstrates the overall performance of the MILP model developed to solve the MPCP. Figure 4.2 presents a few clustering optimal solution examples for $\alpha = 1$. Figure 4.2a presents a solution limited to 80 km of maximum intra-cluster distance. It can be noted that many clusters have their platforms considerably distant from each other, such as clusters C4 and C8. In Figure 4.2b, there are more clusters with smaller distances among their platforms. Clusters C1, C8, and C14 are pertinent examples of this new configuration. However, there still are clusters whose platforms' locations are relatively scattered, such as C6 and C12. In fact, this can happen for example due to the existence of constraints related to vessel's capacity and night-shift limitations, which can impose alternative cluster configurations.



(a) Max. intra-cluster distance: 80 km.

(b) Max. intra-cluster distance: 45 km.

Figura 4.2: Example of clustering solutions for $\alpha = 1$.

With a few solution examples introduced, now the overall results obtained from applying the model to the 60 instances are presented. Table 4.2 reveals the model's performance to solve instances that result in MILP models with 30 non-negative real variables, 111070 binary variables, and 217413 constraints. In that table, #ins, #opt, and #tlf stand for number of instances, number of instances deemed optimal, and number of time limit feasible (sub-optimal) instances, respectively.

Tabela 4.2: Overall performance of the MILP model to solve the MPCP.

#ins	#opt	time (s)	#tlf	gap (%)
60	22	250.4	38	2.7

Table 4.2 shows that only 37% of the instances were solved to optimality within 250.4 seconds, on average. Despite not achieving optimality in 63% of the instance set, the quality of the sub-optimal solutions is good, as their average gap is 2.7%, obtained within 3600 seconds set as run-time limit.

4.2.3 Effect of the composite objective function

Table 4.3 presents results regarding the impact of the composite objective function for instances pivoted by α. Among the optimal solutions, it can noted that higher α-values tend to promote more optimal solutions with significant run-time reduction. An opposite behavior passes for time limit feasible instances.

Tabela 4.3: Composite objective function's effect on model's performance for instances pivoted by α.

α	#ins	#opt	time (s)	#tlf	gap (%)
0	12	1	902.2	11	4.8
0.25	12	–	–	12	2.8
0.5	12	1	203.4	11	1.2
0.75	12	8	505.3	4	0.7
1	12	12	30.0	–	–
Total	60	22	250.4	38	2.7

For $\alpha = 0$, a single instance took roughly 10 minutes to be solved, whereas for $\alpha = 1$ all of them were solved on average in 0.5 minute, one order of magnitude less. As a consolidated view, optimizing an objective function that minimizes only the number of clusters, achieved when $\alpha = 1$, requires less computational efforts than minimizing solely an upper bound for all clusters' berth times, given by $\alpha = 0$. In other words, the model revealed to be more tractable for $\alpha = 1$.

Table 4.4 presents clustering results. In that table, “opt"and “tlf"stand for optimal and time limit feasible groups of results, respectively. The group of results under “opt"(“tlf") in that table refers to the optimal (time limit feasible) results in

Table 4.3. The columns named n_{clu}, n_{plat}, and bt signify number of clusters, platforms per cluster, and berth time, respectively, all of them related to their final solutions.

Tabela 4.4: Composite objective function's effect on clustering results for instances pivoted by α. † Arithmetic mean.

α	opt			tlf		
	$n^\dagger_{\text{clu}}$	$n^\dagger_{\text{plat}}$	$bt^\dagger$ (h)	$n^\dagger_{\text{clu}}$	$n^\dagger_{\text{plat}}$	$bt^\dagger$ (h)
0	29.00	3.00	5.13	29.00	3.00	5.38
0.25	–	–	–	21.58	4.04	7.14
0.5	22.00	3.95	6.74	20.00	4.40	7.80
0.75	20.00	4.39	7.67	20.00	4.39	7.90
1	20.00	4.39	8.60	–	–	–
Total	20.50	4.31	8.02	23.11	3.88	6.90

From Table 4.4, one can note that setting $\alpha = 0$ leads to solutions with the maximum number of clusters, i.e., $n_{\text{clu}} = 29$, which precisely is the PSV's fleet size made available for the 87 maritime platforms of each instance. In other words, fixing α at zero communicates to the model no concern regarding the number of clusters in the final solution, which in turn leads to designating as less platforms as possible per vessel to achieve the smallest berth times. This is demonstrated by the average number of platforms $n_{\text{plat}} = 3.00$, which leaves constraints 4.3 active in the final solution, and by the average berth time, corresponding to $bt = 5.13$ hours for optimal solutions, and to $bt = 5.38$ hours for sub-optimal ones (time limit feasible).

In the other extreme, i.e., $\alpha = 1$, the minimum number of clusters is obtained, $n_{\text{clu}} = 20.00$, on average among all instances. This means that there are more platforms per clusters and, consequently, longer berth times, which is explained by the increase in the number of platforms per cluster, $n_{\text{plat}} = 4.39$, on average, indicating the existence of some clusters with 5 platforms, and by $bt = 8.60$ hours. For $\alpha \in \{0.25, 0.5, 0.75\}$, the conflict between the two objectives, i.e., minimizing the number of clusters and associated berth times, turns more evident, given the existence of a tendency indicating that the average number of clusters reduces at the cost of increasing the average berths times.

4.3 Discussion on the practical application

The MILP model designed to solve instances of the MPCP consists of a suitable mathematical representation of various real aspects inherent to the problem, which are important to be considered jointly in decisions related to clustering of platforms. The model allows one to solve 37% of the instances designed optimally within the relative optimality gap value of 0.5% (the tolerance specified) in 4.2 minutes, on average. For the remainder 63% of the instances, the model produced good quality solutions, what is demonstrated by their average relative gap value of 2.7%, obtained within 1h (run-time limit). In practice, these numbers indicate that it would be reasonable to use such a model in real life operations. These results also pave path for redesigning clusters on a shorter term, e.g., a week ahead, which can be advantageous to reduce deviations between the logistic service plan and the offshore operational context.

Another relevant standpoint to interpret the results obtained resides in what value to choose for α, in practice. Typically, it is desirable to concomitantly minimize both quantities: number of clusters and berth times. The first leads to hire less transportation capacity on the medium term, assuming one vessel is enough, in cargo capacity per route, to attend a cluster. The second results in less demand for supply base berths, what in turn can promote cost reductions for avoiding to charter extra berth time. A decision that integrates these benefits mentioned and, in the scope of the instances employed, significantly assures optimality can be made for $\alpha = 0.75$. This is demonstrated by: (i) Table 4.3 – given that 67% (8 out of 12) of the instances for $\alpha = 0.75$ were solve optimally, and the 33% of sub-optimal instances presented low average gap of 0.7% – and (ii) Table 4.4, given that the number of clusters and platforms per cluster achieves their best values, i.e., $n_{\text{clu}} = 20.00$ and $n_{\text{plat}} = 4.39$, on average.

Capítulo 5

Routing and scheduling of platform supply vessels

This chapter contains the mathematical modeling of the d-PSVRSP. Besides, it also presents a series of experimental results obtained from solving artificial instances inspired from real life data of an oil and gas operator that develops its offshore logistics activities in southeast Brazilian waters, and a discussion on the practical application of the method proposed.

5.1 Mathematical modeling

This section presents an MILP formulation for the d-PSVRSP. Firstly, the formulation does not regard the problem as a periodic one in the sense of fixing *a priori* the number of visits that a platform can have within a pre-specified time horizon or even within a trip. The periodic aspect in the present problem relies only on the fact that each cluster's platforms place cargo delivery and pickup orders daily, which in turn are attended from a fixed, regular, and previously defined scheme of PSV daily departures from the supply base.

Secondly, the formulation proposed actually routes PSVs to “visit"cargo orders, which means that when a vessel arrives at a platform, it does not necessarily have to fulfill all orders requested by such a platform at a single visit. As a platform and the orders placed by it share the same location coordinates, the sequence of platforms visited implicitly results from the routing solutions found with the MILP model for the d-PSVRSP. Figure 1.2b illustrates a few routing examples that consider orders as visit nodes, instead of platforms themselves. The formulation also allows multiple visits at a platform per trip, with a same vessel or with a different one. This can achieved due to the fact that the nodes visited by a PSV are actually orders, instead of platforms

At last, the formulation allows to generate routing solutions in which a PSV can be scheduled for two or more trips ahead. Solutions like that are particularly useful when there is not enough transportation capacity to cope with the cargo demand placed by the platforms of cluster in a single trip. Before presenting the MILP formulation, additional notation is introduced for convenience.

5.1.1 Specific notation

This section presents the notation necessary specifically for the d-PSVRSP. Let a set of supply base copies for vessel $k \in \mathcal{V}$ be $\mathcal{N}_0^k := \left\{h^k+1, h^k+2, ..., h^{k+1}\right\}$, in which $h^k := n + \sum_{k'=1}^{k-1}\left(L^{k'}+1\right)$, n is the total number of orders, and L^k is the maximum number of consecutive trips allowed for k.

Sets

$\mathcal{C}$	Set of maritime platforms, defined as: $\mathcal{C} := \{1, 2, \ldots, b\}$.
$\mathcal{P}$	Set of commodity types.
$\mathcal{P}_- \subseteq \mathcal{P}$	Set of commodity types that are ready to be picked.
$\mathcal{P}_+ \subseteq \mathcal{P}$	Set of commodity types that are ready to be delivered.
$\mathcal{O}$	Set of orders requested by maritime platforms, defined as: $\mathcal{O} := \{1, 2, \ldots, n\}$.
$\mathcal{O}_-$	Set of pickup orders, defined as: $\mathcal{O}_- := \{i \in \mathcal{O} : p^i \in \mathcal{P}_-\}$.
$\mathcal{W}_i$	Set of time windows for order $i \in \mathcal{O}$ requested by platform $c^i \in \mathcal{C}$.
$\mathcal{V}$	Set of PSVs.
$\mathcal{Q}$	Set of compartment types for a vessel.
$\mathcal{P}_q \subseteq \mathcal{P}$	Set of commodity types compatible with compartment type $q \in \mathcal{Q}$.
$\underline{\mathcal{N}}_0^k$	Set of supply base copies for vessel $k \in \mathcal{V}$, except the first supply base copy, defined as: $\underline{\mathcal{N}}_0^k := \mathcal{N}_0^k \setminus \left\{h^k+1\right\}$
$\overline{\mathcal{N}}_0^k$	Set of supply base copies for vessel $k \in \mathcal{V}$, except the last supply base copy, defined as: $\overline{\mathcal{N}}_0^k := \mathcal{N}_0^k \setminus \left\{h^{k+1}\right\}$.
$\underline{\overline{\mathcal{N}}}_0^k$	Set of supply base copies for vessel $k \in \mathcal{V}$, except the first and last supply base copies, defined as: $\underline{\overline{\mathcal{N}}}_0^k := \underline{\mathcal{N}}_0^k \cap \overline{\mathcal{N}}_0^k$.
$\underline{\mathcal{N}}^k$	Set formed by the union of the set of orders and set $\underline{\mathcal{N}}_0^k$, defined as: $\underline{\mathcal{N}}^k := \mathcal{O} \cup \underline{\mathcal{N}}_0^k$.
$\overline{\mathcal{N}}^k$	Set formed by the union of the set of orders and set $\overline{\mathcal{N}}_0^k$, defined as: $\overline{\mathcal{N}}^k := \mathcal{O} \cup \overline{\mathcal{N}}_0^k$.
$\mathcal{A}^k$	Set of arcs for vessel $k \in \mathcal{V}$, defined as: $\mathcal{A}^k := \left\{(i,j) \in \overline{\mathcal{N}}^k \times \underline{\mathcal{N}}^k : i \neq j, i \neq h^{k+1}, j \neq h^k+1\right\} \setminus \left\{(i,j) \in \overline{\mathcal{N}}_0^k \times \underline{\mathcal{N}}_0^k : j \geq i+2\right\}$.
$\mathcal{A}$	Set of arcs, defined as: $\mathcal{A} := \bigcup_{k \in \mathcal{V}} \mathcal{A}^k$.

$\mathcal{A}_0^k$	Set of supply base arcs for vessel $k \in \mathcal{V}$, defined as: $\mathcal{A}_0^k := \Big\{(i,j) \in \overline{\mathcal{N}}_0^k \times \underline{\mathcal{N}}_0^k : j = i+1\Big\}$.
$\mathcal{A}_0$	Set of supply base arcs, defined as: $\mathcal{A}_0 := \bigcup_{k \in \mathcal{V}} \mathcal{A}_0^k$.
$\tilde{\mathcal{A}}$	Set arcs from pickup to delivery orders requested by the same platform, defined as: $\tilde{\mathcal{A}} := \{(i,j) \in \mathcal{O}_- \times \mathcal{O}_+ : q \in \mathcal{Q}, p^i, p^j \in \mathcal{P}_q, c^i, c^j \in \mathcal{C}, c^i = c^j\}$.
$\mathcal{T}^k$	Set of tuples for which there will be no commodity flow performed by vessel $k \in \mathcal{V}$, defined as: $\mathcal{T}^k := \Big\{(i,j,p) \in \overline{\mathcal{N}}_0^k \times \mathcal{O} \times \mathcal{P}_-\Big\} \bigcup \big\{(i,j,p) \in \mathcal{O} \times \underline{\mathcal{N}}_0^k \times \mathcal{P}_+\big\}$.
$\mathcal{T}$	Set of tuples for which there will be no commodity flow performed, defined as: $\mathcal{T} := \bigcup_{k \in \mathcal{V}} \mathcal{T}^k$.

Parameters

ϕ_p	Loading/unloading rate of commodity $p \in \mathcal{P}$ at a platform.
σ_p	Loading/unloading rate of commodity $p \in \mathcal{P}$ at the supply base.
B_{ip}	Pickup quantity of commodity $p \in \mathcal{P}$, for order $i \in \mathcal{O}$.
L_{ip}	Delivery quantity of commodity $p \in \mathcal{P}$, for order $i \in \mathcal{O}$.
D_i	Quantity of commodity $p^i \in \mathcal{P}$ for order $i \in \mathcal{O}$ to be either delivered to or picked up at platform $c^i \in \mathcal{C}$.
ST_i	Service time for order $i \in \mathcal{O}$.
ET_{ih}	Earliest time point for time window $h \in \mathcal{W}_i$ for order $i \in \mathcal{O}$.
LT_{ih}	Latest time point for time window $h \in \mathcal{W}_i$ for order $i \in \mathcal{O}$.
ζ_i	Penalty per hour for violating ET_{ih}, $i \in \mathcal{O}$, $h \in \mathcal{W}_i$.
β_i	Penalty per hour for violating LT_{ih}, $i \in \mathcal{O}$, $h \in \mathcal{W}_i$, defined as: $\beta_i = 2\zeta_i$.
DT_i	Due time for pickup order $i \in \mathcal{O}_-$, by which the task of unloading this order onto the onshore base has to be finished.
Q_q^k	Capacity of compartment type $q \in \mathcal{Q}$ for vessel $k \in \mathcal{V}$.
AT^k	Moment at which vessel $k \in \mathcal{V}$ becomes available for loading at the base.
N_{ij}^k	Navigation time for vessel $k \in \mathcal{V}$ from c^i to c^j, for $c^i, c^j \in \mathcal{C}$, $i,j \in \mathcal{O} \cup \{0\}$, $i \neq j$.
SE^k	Setup time for vessel $k \in \mathcal{V}$ before it departures from a platform.
SP^k	Safe positioning time for vessel $k \in \mathcal{V}$ when arriving at a platform.
T_{ij}^k	Total travel time for vessel $k \in \mathcal{V}$ from c^i to c^j, $c^i, c^j \in \mathcal{C}$, $i,j \in \mathcal{O} \cup \{0\}, i \neq j$, defined in 3.2.
L^k	Maximum number of consecutive trips allowed for vessel $k \in \mathcal{V}$.
TD^k	Maximum trip duration for vessel $k \in \mathcal{V}$.
θ^k	Fuel cost per hour for $k \in \mathcal{V}$ awaiting at the base's vicinity.
φ^k	Fuel cost per hour for $k \in \mathcal{V}$ operating at the supply base.
γ^k	Fuel cost per hour for $k \in \mathcal{V}$ in navigation.

δ^k Fuel cost per hour for $k \in \mathcal{V}$ enrolled in service-related tasks offshore, such as cargo handling, waiting times, or setups.

ξ^k Non-dimensional, positive factor devised for each $k \in \mathcal{V}$ to promote the use of smaller vessels.

α Weight parameter, defined as: $\alpha \in [0, 1]$.

5.1.2 Modeling

This section presents an MILP formulation for the d-PSVRSP. The constraints and objective function of such a formulation are introduced in “blocks". The first block presents the degree constraints, which produce the route's structure. The second block accounts for the commodity flow constraints. The third block defines constraints that express time moments. The fourth block introduces constraints that impose a maximum route duration. The fifth block of constraints serves the purpose of avoiding service times to overlap for each platform's orders. The sixth block of constraints accounts for the time windows to be used. The seventh block of constraints defines the due time (deadline) for each pickup order. The last two blocks of constraints express important routes' time components, such as travelling, service, berth times, cumulative full consumption, and utilization time. The model's objective function is presented in the last block.

Nodes' degree constraints. Let a binary variable x_{ij}^k be 1 if and only if vessel $k \in \mathcal{V}$ traverses arc $(i, j) \in \mathcal{A}^k$. Constraints (5.1) and (5.2) enforce that every used vessel must start a trip from the supply base and return back to it after finishing its offshore agenda. For convenience, a vessel $k \in \mathcal{V}$ is enforced to perform a fictitious trip from supply base node $i \in \overline{\mathcal{N}}_0^k$ to node $i + 1 \in \underline{\mathcal{N}}_0^k$ even if this vessel is not used for its l-th trip. Constraints (5.3) simply state the degree relationship at each order node, while constraints (5.4) ensure that every order will be served exactly once. Constraints (5.5) enforce that each vessel must use its supply base nodes sequentially.

$$\sum_{(i,j)\in\mathcal{A}^k} x_{ij}^k = 1 \qquad \forall k \in \mathcal{V}, \forall i \in \overline{\mathcal{N}}_0^k \tag{5.1}$$

$$\sum_{(j,i)\in\mathcal{A}^k} x_{ji}^k = 1 \qquad \forall k \in \mathcal{V}, \forall i \in \underline{\mathcal{N}}_0^k \tag{5.2}$$

$$\sum_{(j,i)\in\mathcal{A}^k} x_{ji}^k = \sum_{(i,j)\in\mathcal{A}^k} x_{ij}^k \qquad \forall k \in \mathcal{V}, \forall i \in \mathcal{O} \tag{5.3}$$

$$\sum_{k\in\mathcal{V}} \sum_{(i,j)\in\mathcal{A}^k} x_{ij}^k = 1 \qquad \forall i \in \mathcal{O} \tag{5.4}$$

$$\sum_{(i,j)\in\mathcal{A}^k} x_{ij}^k \leqslant \sum_{(j,i)\in\mathcal{A}^k} x_{ji}^k \qquad \forall k\in\mathcal{V}, \forall i\in\overline{\underline{\mathcal{N}}}_0^k \tag{5.5}$$

Commodity flow constraints. Let a non-negative variable z_{ijp} represent the amount of commodity $p\in\mathcal{P}$ flowing over arc $(i,j)\in\mathcal{A}$. Constraints (5.6) ensure that compartment capacity constraints are respected, while constraints (5.7) enforce the demand balance at each order node. The notation $\mathbb{1}_{p=p^i}$ is a indicator function, resulting 1, if $p=p^i$, or 0, otherwise. For convenience, if $i\in\mathcal{O}_-$, i.e., i is a pickup order, let D_i take a negative value. Constraints (5.8) enforce that there are no pickups leaving the supply base, nor deliveries returning to it, respectively.

$$\sum_{p\in\mathcal{P}_q} z_{ijp} \leqslant \sum_{k\in\mathcal{V}} Q_q^k x_{ij}^k \qquad \forall (i,j)\in\mathcal{A}\setminus\mathcal{A}_0, \forall q\in\mathcal{Q} \tag{5.6}$$

$$\sum_{(j,i)\in\mathcal{A}} z_{jip} - \sum_{(i,j)\in\mathcal{A}} z_{ijp} = \mathbb{1}_{p=p^i} D_i \sum_{k\in\mathcal{V}} \sum_{(i,j)\in\mathcal{A}^k} x_{ij}^k \qquad \forall i\in\mathcal{O}, \forall p\in\mathcal{P} \tag{5.7}$$

$$z_{ijp} = 0 \qquad \forall (i,j,p)\in\mathcal{T} \tag{5.8}$$

Time constraints. Let s_i^k be a non-negative variable that represents the start time of the cargo loading operation for vessel $k\in\mathcal{V}$ at its supply base node $i\in\overline{\mathcal{N}}_0^k$. Let a_i and d_j be non-negative variables that denote the arrival time at node $i\in\underline{\mathcal{N}}^k$ and departure time from node $j\in\overline{\mathcal{N}}^k$, respectively. Let $\tilde{a}_{ij}$ be a non-negative variable that represents the arrival time at node j from node i if arc $(i,j)\in\mathcal{A}$ is traversed by some vessel and 0, otherwise. A non-negative variable f_i^k is defined to represent the moment at which vessel $k\in\mathcal{V}$ finishes its unloading operation at its supply base node $i\in\underline{\mathcal{N}}_0^k$. Constraints (5.9) state that vessel k is available from the moment AT^k. Constraints (5.10) define the vessel departure time from the supply base. Together, constraints (5.11) – (5.13) define the arrival time for a vessel at an order node or at a supply base node. The big-M value M_{ij} is properly defined in the Appendix. Constraints (5.14) assure that the order service time is respected. Constraints (5.15) enforce that every vessel must return to the supply base and unload onto it pickups collected during the trip. Constraints (5.16) assure that a vessel can not start its next trip before the previous one is finished. The subtour elimination is jointly assured from constraints 5.10 – 5.15, as they impose that a vessel used must start and finish its route at the supply base.

$$s_{h^k+1} \geqslant AT^k \qquad \forall k\in\mathcal{V} \tag{5.9}$$

$$s_i + \sum_{p\in\mathcal{P}_+} \sum_{(i,j)\in\mathcal{A}^k\setminus\mathcal{A}_0^k} \sigma_p z_{ijp} = d_i \qquad \forall k\in\mathcal{V}, \forall i\in\overline{\mathcal{N}}_0^k \tag{5.10}$$

$$0 \leq \tilde{a}_{ij} \leqslant M_{ij} \sum_{k\in\mathcal{V}:(i,j)\in\mathcal{A}^k} x_{ij}^k \qquad \forall (i,j)\in\mathcal{A} \tag{5.11}$$

$$d_i + \sum_{k\in\mathcal{V}} \sum_{(i,j)\in\mathcal{A}^k} T_{ij}^k x_{ij}^k \leqslant \sum_{(i,j)\in\mathcal{A}} \tilde{a}_{ij} \qquad \forall i \in \bigcup_{k\in\mathcal{V}} \overline{\mathcal{N}}^k \tag{5.12}$$

$$a_i = \sum_{(j,i)\in\mathcal{A}} \tilde{a}_{ji} \qquad \forall i \in \bigcup_{k\in\mathcal{V}} \underline{\mathcal{N}}^k \tag{5.13}$$

$$a_i + ST_i = d_i \qquad \forall i \in \mathcal{O} \tag{5.14}$$

$$a_i + \sum_{p\in\mathcal{P}_-} \sum_{(j,i)\in\mathcal{A}^k\setminus\mathcal{A}_0^k} \sigma_p z_{jip} = f_i \qquad \forall k\in\mathcal{V}, \forall i \in \underline{\mathcal{N}}_0^k \tag{5.15}$$

$$f_i \leqslant s_i \qquad \forall k\in\mathcal{V}, \forall i \in \overline{\mathcal{N}}_0^k \tag{5.16}$$

Trip duration limit constraints. Constraints (5.17) apply a limitation on the trip time per vessel.

$$f_{i+1} - s_i \leq TD^k \qquad \forall k\in\mathcal{V}, \forall i \in \overline{\mathcal{N}}_0^k \tag{5.17}$$

Service precedence constraints. Let a binary variable y_{ij} be 1 if the service associated with order $i \in \mathcal{O}$ finishes before that of order $j \in \mathcal{O}$ starts, and 0 otherwise. Constraints (5.18) – (5.19) enforce non-overlapping services for orders that belong to the same platform. Constraints (5.20) assure that pickup orders be serviced before the delivery ones.

$$d_j \leqslant a_i + DT_j y_{ij} \qquad \forall (i,j)\in\mathcal{A}\setminus\mathcal{A}_0, \forall c\in\mathcal{C} : c^i = c^j \tag{5.18}$$

$$d_i \leqslant a_j + DT_i\,(1-y_{ij}) \qquad \forall (i,j)\in\mathcal{A}\setminus\mathcal{A}_0, \forall c\in\mathcal{C} : c^i = c^j \tag{5.19}$$

$$y_{ij} = 1 \qquad \forall (i,j)\in\tilde{\mathcal{A}} \tag{5.20}$$

Time window constraints. Let a binary variable u_{ih} be 1 if and only if order $i \in \mathcal{O}$ is served within time window $h \in \mathcal{W}_i$. Constraints (5.21) compel the model to select only one time window for each order. Constraints (5.22) – (5.23) assure that the offshore service time of an order must happen within the time window selected for that order.

$$\sum_{h\in\mathcal{W}_i} u_{ih} = 1 \qquad \forall i \in \mathcal{O} \tag{5.21}$$

$$a_i \geqslant \sum_{h\in\mathcal{W}_i} ET_{ih} u_{ih} \qquad \forall i \in \mathcal{O} \tag{5.22}$$

$$d_i \leqslant \sum_{h \in \mathcal{W}_i} LT_{ih} u_{ih} \qquad \forall i \in \mathcal{O} \tag{5.23}$$

Due time constraints. Let binary variables $\bar{y}_i^k$ be 1 if and only if vessel $k \in \mathcal{V}$ transports order $i \in \mathcal{O}_-$ and $\tilde{y}_{ij}^k$ be 1 if and only if vessel $k \in \mathcal{V}$ transports order $i \in \mathcal{O}_-$ to supply base node $j \in \underline{\mathcal{N}}_0^k$. Constraints (5.24) inform what vessel transports what pickup order. Constraints (5.25) enforce that a vessel in use to transport a pickup order must do it in one of its trips. Constraints (5.26) – (5.29) account for the selection of a trip for a vessel to transport pickup orders back to the base. Constraints (5.30) impose the due time for a pickup order carried by certain vessel. The big-M values $M_{2,i}$, M_3^k, $M_{4,i}$, and M_5^k are properly defined in the Appendix.

$$\bar{y}_i^k = \sum_{(i,j) \in \mathcal{A}^k \setminus \mathcal{A}_0} x_{ij}^k \qquad \forall k \in \mathcal{V}, \forall i \in \mathcal{O}_- \tag{5.24}$$

$$\sum_{j \in \underline{\mathcal{N}}_0^k} \tilde{y}_{ij}^k = \bar{y}_i^k \qquad \forall k \in \mathcal{V}, \forall i \in \mathcal{O}_- \tag{5.25}$$

$$a_i - a_{h^k+2} \leqslant M_{2,i} \left(1 - \tilde{y}_{i,h^k+2}^k\right) \qquad \forall k \in \mathcal{V}, \forall i \in \mathcal{O}_- \tag{5.26}$$

$$d_j - a_i \leqslant M_3^k \left(1 - \tilde{y}_{ij}^k\right) \qquad \forall k \in \mathcal{V}, \forall i \in \mathcal{O}_-, \forall j \in \overline{\underline{\mathcal{N}}}_0^k \tag{5.27}$$

$$d_i - a_{j+1} \leqslant M_{4,i} \left(1 - \tilde{y}_{ij}^k\right) \qquad \forall k \in \mathcal{V}, \forall i \in \mathcal{O}_-, \forall j \in \overline{\underline{\mathcal{N}}}_0^k \tag{5.28}$$

$$d_{h^{k+1}-1} - a_i \leqslant M_3^k \left(1 - \tilde{y}_{i,h^{k+1}}^k\right) \qquad \forall k \in \mathcal{V}, \forall i \in \mathcal{O}_- \tag{5.29}$$

$$f_j^k \leqslant DT_i + M_5^k \left(1 - \tilde{y}_{ij}^k\right) \qquad \forall k \in \mathcal{V}, \forall i \in \mathcal{O}_-, \forall j \in \underline{\mathcal{N}}_0^k \tag{5.30}$$

Activity type constraints. Let the following non-negative variables be for vessel $k \in \mathcal{V}$: q^k, g^k, t^k, and v^k which represent the total supply base service time, total offshore time, total navigation time, and total service time, respectively. Constraints (5.31) express the total supply base service time per vessel, which includes loading delivery orders before trip's departure and unloading onto the base pickups the collected offshore. Constraints (5.32) capture the total time a vessel stays offshore. Constraints (5.33) address the total navigation time for each vessel. Constraints (5.34) express the total time a vessel is enrolled in service related procedures.

$$q^k = \sum_{i=h^k+1}^{h^{k+1}-1} \left(d_i - s_i^k\right) + \sum_{i=h^k+2}^{h^{k+1}} \left(f_i^k - a_i\right) \qquad \forall k \in \mathcal{V} \tag{5.31}$$

$$g^k = \sum_{i=h^k+1}^{h^{k+1}-1} \left(a_{i+1} - d_i\right) \qquad \forall k \in \mathcal{V} \tag{5.32}$$

$$t^k = \sum_{(i,j) \in \mathcal{A}^k} N_{ij}^k x_{ij}^k \qquad \forall k \in \mathcal{V} \tag{5.33}$$

$$v^k = g^k - t^k \qquad \forall k \in \mathcal{V} \tag{5.34}$$

Fuel consumption and vessel utilization constraints. Let the non-negative variables f_1 and f_2 be the marginal cumulative vessels' fuel cost and the cumulative vessels' utilization time, respectively. Equations (5.35) specify f_1. Equations (5.36) specify f_2.

$$f_1 = \sum_{k \in \mathcal{V}} \left(\varphi^k - \theta^k\right) q^k + \left(\gamma^k - \theta^k\right) t^k + \left(\delta^k - \theta^k\right) v^k \tag{5.35}$$

$$f_2 = \sum_{k \in \mathcal{V}} \xi^k \left(f^k_{h^{k+1}} - AT^k\right) \tag{5.36}$$

Objective function. The problem's minimization objective is presented in (5.37), which is a composite function formulated from a weighed sum of f_1 and f_2 for $\alpha \in [0, 1]$. The parameter η, defined in the Appendix, specifically in 7.2, monetizes f_2 so that both f_1 and f_2 keep commensurable.

$$\operatorname*{Min}_{f_1, f_2} \alpha f_1 + (1 - \alpha)\, \eta f_2 \tag{5.37}$$

The goal of the d-PSVRSP is to perform the minimization expressed in (5.37), including decision variables subject to constraints (5.1) – (5.36).

5.1.3 Strengthening inequalities

The essence of the algorithm used to separate[1] strengthening inequalities for the d-PSVRSP is reproduced here, aiming to better clarify the reader on it. The strengthening inequalities are an adaptation of the classic *rounded capacity inequalities* (RCIs) LAPORTE and NOBERT (1983). Specifically, RCIs are defined as follows:

$$\sum_{k \in \mathcal{V}} \sum_{i \notin \mathcal{S}} \sum_{j \in \mathcal{S}} \left\lceil \frac{Q^k_q}{\overline{Q}_q} \right\rceil x^k_{ij} \geqslant \left\lceil \frac{\sum_{i \in \mathcal{S}} \sum_{p \in \mathcal{P}_q} \mathbb{1}_{p=p^i} D_i}{\overline{Q}_q} \right\rceil \qquad \forall \mathcal{S} \subseteq \mathcal{O}, \forall q \in \mathcal{Q}, \tag{5.38}$$

[1]The development of the separation algorithm for strengthening inequalities for the d-PSVRSP is not one of the dissertation's contributions. The algorithm was developed by researchers of the Center for Advanced Process Decision-making (CAPD), at Carnegie Mellon University (CMU), PA, EUA. The algorithm is just utilized in this dissertation, as fruit of a joint research period between CMU and Petrobras, in which this dissertation's author developed the MILP model for the d-PSVRSP.

where $\overline{Q}_q := \max_{k \in \mathcal{V}} Q_q^k$. RCIs of this form can be considered as rounding inequalities with $\overline{Q}_q$ being the division term before the rounding procedure, thus they are valid inequalities.

In order to separate RCIs, it is first constructed the support graph $\overline{G}$ that corresponds to the LP fractional solution $\overline{x}$ at a given branch-and-bound node. The set of supply base copies, $\cup_{k \in \mathcal{V}} \mathcal{N}_0^k$, is then shrunk to be a single vertex in the graph and then assign $\sum_{k \in \mathcal{V}} \left\lceil \frac{Q_q^k}{\overline{Q}_q} \right\rceil x_{ij}^k \left(\sum_{p \in \mathcal{P}_q} \mathbb{1}_{p=p^i} D_i / \overline{Q}_q \right)$ as the weight for arc (i, j) (node i). Three heuristics are adopted to separate RCIs.

Specifically, as LYSGAARD *et al.* (2004) suggested, firstly it is identified all *connected components* in the graph $\overline{G}$ as candidates. The second heuristics separates the so-called *fractional capacity inequalities* via the max-flow algorithm. If both heuristics fail, then it is used the *tabu search* method proposed by AUGERAT *et al.* (1998) to identify violated RCIs. Readers are referred to AUGERAT *et al.* (1998) and WANG *et al.* (2021) for implementation details.

5.2 Computational studies

In this section, the instances' generation process is described, as well as the computational results obtained from the approach proposed to solve the d-PSVRSP. Implementations are made in C++ and all subordinate linear and mixed-integer linear programs were solved using the CPLEX Optimizer 12.9.0[2] through the C application programming interface, with all settings being default, except the relative optimality gap tolerance and run-time limit, set as: `MIPGap = 0.1%` and `TimeLimit = 1800` seconds. The gap value reported by the solver is defined as: $gap = \frac{UB - LB}{UB} \times 100$, in which *UB* stands for "upper bound"and *LB* stands for "lower bound".

All computations were performed on an Intel Xeon CPU E5-2689 v4 server running at 3.10 GHz. A total of 128GB of available RAM was shared among 19 copies of the model running in parallel on the server. Each instance was solved by one copy of the model using a single thread. All CPU times and relative optimality gap values presented as results in this section are calculated as arithmetic means.

5.2.1 Benchmark instances

The platform locations are illustrated in the Figure 5.1. It is considered a group of 13 platforms, the solid dots in that figure, to be used in the design of benchmark instances. Some descriptive statistics about this group are: the closest platform

[2]The CPLEX solver was a natural choice because the routines to integrate the RCI cuts into the solver's solution process were already implemented for CPLEX by the CMU's research group, as explained in section 5.1.3.

from the supply base is located at 132 km and the farthest at 191 km. Among the platforms, the minimum distance is 1.65 km and, the maximum, 61 km.

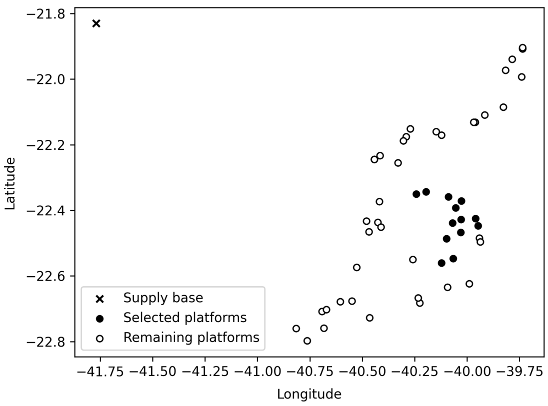


Figura 5.1: Platform and supply base locations.

The instances are restricted to orders of deck cargo, diesel, and water, which are devised per platform from operator's historical data. For convenience, pickup and delivery orders of deck cargo are regarded as distinct commodities, whereas diesel and water appear only as deliveries, such as typically seen in practice. Three PSV sizes are used in the instances: small (`S`), medium (`M`), and large (`L`). Vessel navigation times are calculated using geodesic distances among the platforms and historical velocities developed in real operations.

The test set for the d-PSVRSP contains 3600 feasible instances designed as follows. There are 60 basic instances whose platforms were randomly selected among the solid dots in Figure 5.1. This selection is made in such a manner that all platforms appear at least once in the basic group. Half of this group is made of instances with 6 platforms each and the remainder half with 7.

Each of the so generated 30-size group contains 6 subgroups of 5 instances each. The first subgroup is formed only with 10-order size instances; the second subgroup, only 12-order size instances; the third, 14-order; and so on up to 20 orders. From the configuration stated, it is possible to arbitrarily classify the basic instance set into sizes referenced by the number of orders: small, 10 – 12 orders; medium, 14 – 16 orders; and large size, 18 – 20.

The basic 60 instances are then branched into 12 versions by varying the number of PSVs according to fleet profiles containing small, medium, and large vessels. Such fleet profiles are organized as follows: 5 versions with 2 vessels each, given by `SS`, `SM`, `MM`, `ML`, `LL`; 5 versions with 3 vessels each, represented by `SSM`, `SMM`, `SML`, `MML`, `MLL`; and 2 versions with 4 vessels each, defined by `SSML` and `SMML`. Since the problem's objective is a composite function weighted by α, the final number of instances depends on how many α-values are utilized. In this study, each of the

$60 \times 12 = 720$ instances are solved for all $\alpha \in \{0, 0.25, 0.5, 0.75, 1\}$, so that the influence of α in the solutions can be perceived. Therefore, the complete test set contains $60 \times 12 \times 5 = 3600$ instances.

The naming for each instance involves the number of orders, PSVs, platforms, a realization identifier, and a suffix to let one know in advance about the fleet profile in use. As an example, the name `16n-2k-6c-3r_ML` corresponds to an instance with 16 orders (`n`), 2 PSVs (`k`), 6 platforms (`c`), realization number 3 (`r`), and fleet profile `ML`, which corresponds to a small PSV and another large one, given the existence of 2 vessels in that instance. The realization number serves to differentiate instances with respect to the platforms selected and types of orders, hence, a name like `16n-2k-6c-5r_ML` means a distinct instance realization. The Table 5.2 reproduces how the 3600 are designed in terms of number of platforms, orders, and vessels' fleet. Interested readers may download the applicable data from `http://gounaris.cheme.cmu.edu/datasets/psvrp/`.

Tabela 5.2: Instances' design. ‡Multiplier; †Number of basic instances; *Number of PSV fleet versions.

(a) Total number of instances given α values.

$m^\ddagger$	3600						
$5\times$	$720_{\alpha=0}$			...	$720_{\alpha=1}$		
$12^*\times$	$60^\dagger$	...	$60^\dagger$	...	$60^\dagger$	...	$60^\dagger$

(b) Basic instances' structure.

$m^\ddagger$	$60^\dagger$					
$2\times$	$30_{\mathtt{6c}}$			$30_{\mathtt{7c}}$		
$6\times$	$5_{\mathtt{10n}}$	...	$5_{\mathtt{20n}}$	$5_{\mathtt{10n}}$	...	$5_{\mathtt{20n}}$

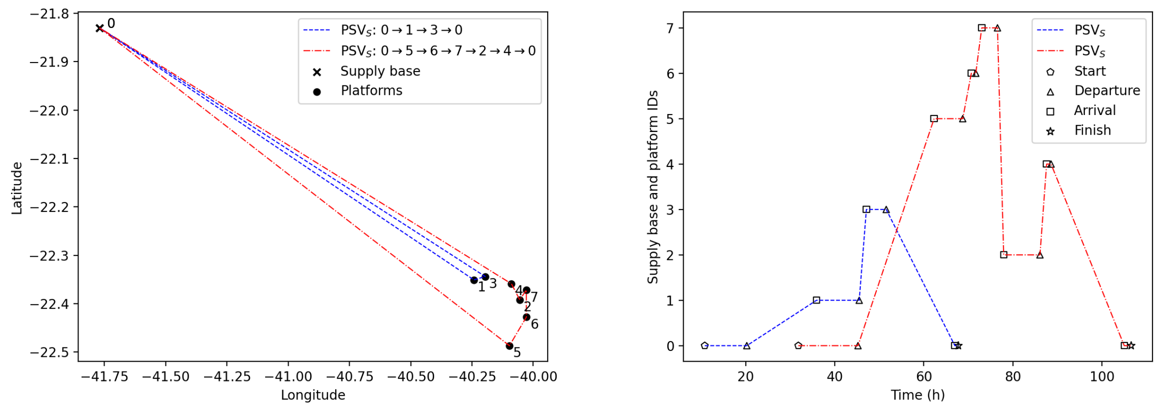
(c) Branching basic instances to assign fleets (PSV sizes).

$12^*\times$							
5			5			2	
`SS`	...	`LL`	`SSM`	...	`MLL`	`SSML`	`SMML`

5.2.2 Performance

This section demonstrates the overall performance of the method developed to solve the d-PSVRSP. However, it is first convenient to depict a few representative solutions. Figure 5.2 presents the optimal routing and scheduling for instance `20n-4k-7c-66r_SSML`. In Figure 5.2a, it can be seen that only two small PSVs are utilized to fulfill orders placed by all 7 platforms. In Figure 5.2b, the time schedule of those PSVs is presented, depicting the route start and finish times as well as arrival and departure times for each platform in the route. The scheduling plot is relevant for providing perspective of how much time each vessel typically spends at each platform or at the supply base.

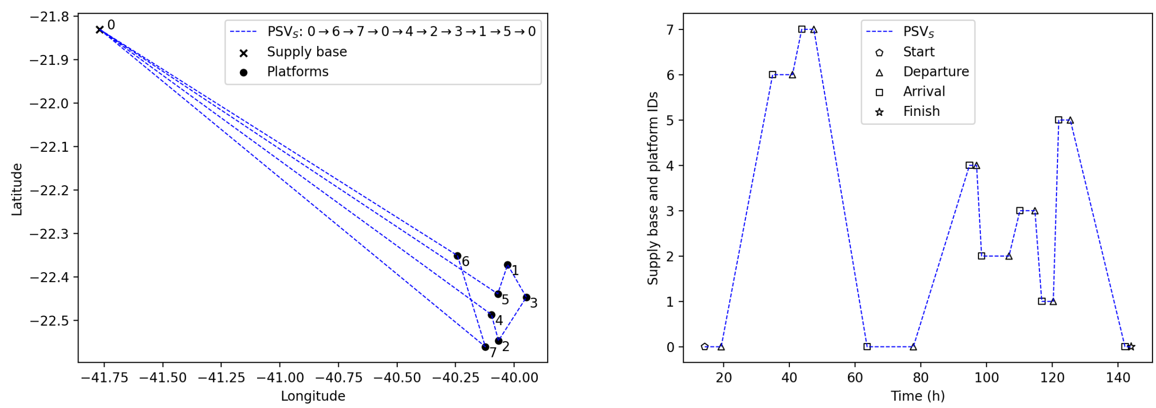
Beyond the traditional routing scheme visualized in Figure 5.2, other solution shapes can also be generated. For example, Figure 5.3a demonstrates an optimal



(a) Routing. (b) Scheduling.

Figura 5.2: Solution shape for instance `20n-4k-7c-66r_SSML`.

routing in which a single small PSV perfoms 2 trips to fully meet the demand of 7 platforms. From the scheduling related to that solution, it is possible to observe that the supply base service occurrences for that vessel, specially the intermediary service between 60 and 80 h, preceding the departure time of the second trip.

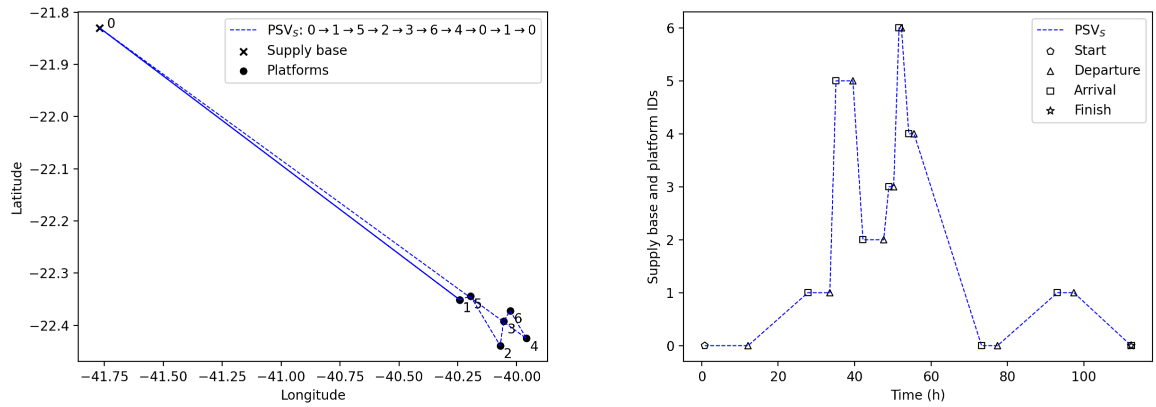


(a) Routing. (b) Scheduling.

Figura 5.3: Solution shape for instance `16n-2k-7c-8r_SM`.

Another example appears in Figure 5.4a. The multi-trip optimal solution again utilizes only one small PSV, yet now the platform with ID 1 has its demand met in 2 trips. The associated scheduling demonstrated in Figure 5.4b reveals the existence of two service occurrences for the platform 1.

It is important to note that the MILP model to solve instances of the d-PSVRSP routes orders, instead of routing platforms, such as traditionally seen in the literature. Hence, in the present problem, the platforms are implicitly routed too, as orders share the same coordinates than their platforms, naturally providing the op-



(a) Routing. (b) Scheduling.

Figura 5.4: Solution shape for instance `14n-2k-6c-22r_SM`.

portunity to have a platform visited more then once in the optimal solution, case this platform has placed more than one order.

With a few example solutions discussed, now the results obtained from the method developed to solve 3600 instances utilizing RCI cuts appear in Table 5.3, which organizes such results by pivoting the instances per number of orders. The instances resulted in MILP models ranging from 666 non-negative real variables, 257 binary variables, and 697 constraints – case of instances with 10 orders and 2 PSVs – to 2691 non-negative real variables, 1843 binary variables, and 2705 constraints – case of instances with 10 orders and 25 scenarios. In Table 5.3, #ord and #ins stand for number of orders and instances, respectively, whereas #opt, #tlf, and #tln stand for number of instances deemed optimal, time limit feasible, and time limit with no incumbent found, respectively.

Tabela 5.3: Results grouped by number of orders.

#ord	#ins	#opt	time (s)	#tlf	gap (%)	#tln
10	600	565	13.5	35	17.8	–
12	600	554	32.7	46	7.0	–
14	600	535	140.4	65	10.5	–
16	600	407	186.4	193	11.7	–
18	600	359	323.5	236	6.9	5
20	600	143	468.1	448	10.7	9
Total	3600	2563	140.5	1023	10.1	14

From 10 to 14 orders per instance, the results reveal a good and consistent performance for the proposed method, since it finds optimal solutions for the vast majority of the instances, taking less than 2.5 minutes for that, on average. From 16 to 18 orders, the performance is still relevant, considering that optimal solutions were found for 68% of the instances with 16 orders within roughly 3 minutes and 60% of those with 18 orders within 5.5 minutes. At 20-order size instances, the method's performance suffers a notable degradation fruit of its poor tractability for larger instances, resulting in only 24% of the them solved to optimality, demanding approximately 8 minutes.

In summary, despite the tractability decreases for 16 orders or more, 71.2% of all instances demonstrate that the method succeeds in finding optimal solutions on average in less than 3 minutes. No optimal solution is found in 28.4% of the instances, which resulted in a low quality demonstrated by the average gap value of 10.1%. From the entire test set, in only 14 instances (less than 0.4%) no incumbent is found.

Table 5.4 presents the results grouped in terms of fleet profile. Among the instances with 2 vessels, the method solves 82% of them in approximately 100 seconds, on average. With 3 vessels, the performance decreases considerably to 67% of the instances solved to optimality, and the associated time increases 60%. With 4 PSVs, 55% of the instances are solved, consuming 3.5 minutes, on average. This table therefore reveals that the problem's tractability is largely worsened as the number of PSVs in the fleet profile increases.

Tabela 5.4: Results grouped by fleet profile.

Fleet profile	#ins	#opt	time (s)	#tlf	gap (%)	#tln
SS	300	277	80.8	23	14.0	–
SM	300	172	148.5	120	11.6	8
MM	300	273	93.6	27	7.3	–
ML	300	241	119.6	58	8.6	1
LL	300	262	89.6	38	5.3	–
2 PSVs	1500	1225	102.7	266	9.8	9
SSM	300	213	240.9	87	5.1	–
SMM	300	163	139.3	134	13.1	3
SML	300	163	193.3	137	14.6	–
MML	300	249	148.3	51	5.5	–
MLL	300	222	100.9	78	7.1	–
3 PSVs	1500	1010	163.2	487	10.3	3
SSML	300	174	237.4	126	5.7	–
SMML	300	154	181.0	144	13.4	2
4 PSVs	600	328	210.9	270	9.8	2
Total	3600	2563	140.5	1023	10.1	14

5.2.3 Effect of the rounded capacity inequalities (RCIs)

Table 5.5 reveals how impacted the method's performance is for utilizing RCI cuts in the solution process. From 10 to 12 orders, i.e., small instances, adding RCIs does not make any difference regarding the number of optimal solutions. In this range, the time decreases at most 40% by adding RCIs. Yet, it was already small, i.e., within 60 seconds when no RCIs are used. Utilizing RCIs starts to demonstrate modest relevance at 14 orders per instance due to an increase of 7.4% in the number of optimal solutions, followed by 35% less time consumed. For 16 orders or more, the gains from utilizing the capacity cuts are clearly distinguishable both in the number of optimal solutions and run-time, since applying the method results in solving at least 38% more instances, accompanied with 60% of reduction in computing time, at least.

Tabela 5.5: Results grouped by number of orders. Effect of the RCI cuts.

#ord	#ins	Without RCI					With RCI				
		#opt	time(s)	#tlf	gap(%)	#tln	#opt	time(s)	#tlf	gap(%)	#tln
10	600	565	19.0	35	17.7	–	565	13.5	35	17.8	–
12	600	554	55.2	46	6.6	–	554	32.7	46	7.0	–
14	600	498	217.2	102	7.9	–	535	140.4	65	10.5	–
16	600	295	452.6	305	7.8	–	407	186.4	193	11.7	–
18	600	75	773.3	524	7.1	1	359	323.5	236	6.9	5
20	600	2	1336.7	588	13.0	10	143	468.1	448	10.7	9
Total	3600	1989	172.8	1600	9.7	11	2563	140.5	1023	10.1	14

Although the overall performance of the method for instances larger than 14 orders is not as good and consistent as for the case with smaller ones, employing such a method makes it possible to solve more than 50% of the instances with 16 and 18 orders in a relatively short time interval. For 20-order size cases, the poor performance is a fact. Yet, solving 143 out of 600, instead of mere 2 in 600, signalizes the cuts grasp some effectiveness at larger instances, therefore motivating further improvements on it. Generally, utilizing RCIs increases 28.8% the number of instances solved, whereas it reduces the run-time in 18.7%. As already seen in section Performance, greater gap values are achieved even for the time limit feasible instances, and in few instances no solution is found.

5.2.4 Effect of the composite objective function

This section presents results obtained from experimenting the solution method, including RCI cuts, with different values of the weighing parameter α, which appears in the objective function expressed in (5.37). By doing this, it is possible to perceive the effect of α over solutions that: (i) seek to minimize purely fuel costs, which is achieved by $\alpha = 1$; (ii) target effective vessel utilization solely, in the case of $\alpha = 0$; (iii) try to balance fuel and vessel utilization from $\alpha \in]0, 1[$.

Considering that PSVs are usually contracted for periods longer than one year, the cost factor on a daily scheduling basis consists only of a variable parcel given by the fuel consumption, which is expressed as the marginal fuel expenditure f_1 in (5.35). It is also relevant to avoid unplanned idleness and meet the demand with smaller PSVs whenever possible, what in turn releases more vessel capacity for upcoming schedules. These two aspects are captured from the variable f_2 in (5.36). Table 5.6 contains a series of results for $\alpha \in \mathcal{H}$, in which $\mathcal{H} := \{0, 0.25, 0.5, 0.75, 1\}$.

Tabela 5.6: Results per alpha value for all fleet profiles and #ord.

α	#ins	#opt	time (s)	#tlf	gap (%)	#tln
0	720	489	141.8	229	9.6	2
0.25	720	500	152.7	217	9.2	3
0.5	720	502	148.4	214	9.9	4
0.75	720	520	148.7	197	10.4	3
1	720	552	112.8	166	11.7	2
Total	3600	2563	140.5	1023	10.1	14

From Table 5.6, it can be noted an increasing trend in the number of optimal solutions (#opt) as α approaches 1, this in turn means obtaining an optimal plan is slightly more tractable and faster when only fuel cost is the minimizing driver in the model's objective function. Besides, no matter the alpha value set, a relatively small solution time is observed. In order to better elucidate the impact of selecting α for the problem studied, Figure 5.5 presents a Pareto-like curve relating scaled, non-dimensional pairs $P(\alpha) = (f_1^{sn}(\alpha), f_2^{sn}(\alpha))$, for $\alpha \in \mathcal{H}$ and instance `14n-2k-6c-8r_ML`. The value on each axis for the coordinates of those pairs are given by:

$$f_1^{sn}(\alpha) = \frac{f_1^*(\alpha)}{\min_{\beta \in \mathcal{H}} \{f_1^*(\beta)\}} \qquad \forall \alpha \in \mathcal{H} \tag{5.39}$$

$$f_2^{sn}(\alpha) = \frac{f_2^*(\alpha)}{\min_{\beta \in \mathcal{H}} \{f_2^*(\beta)\}} \qquad \forall \alpha \in \mathcal{H} \tag{5.40}$$

In (5.39) and (5.40), $f_1^*(\alpha)$ and $f_2^*(\alpha)$ are the optimal values for f_1 and f_2, respectively, obtained for instance `14n-2k-6c-8r_ML` given $\alpha \in \mathcal{H}$. From Figure 5.5, if one takes $\alpha = 1$ as an example, the plot demonstrates f_2^{sn} approaches 1.09, which in practice means the solution for that instance led to f_2 values almost 9% greater than the minimum obtained. In other words, if what matters is just saving fuel, it comes as a trade-off with awaiting more to have vessels available for future scheduling requests.

Now looking at the other extreme of the Pareto plot for that instance, it is possible to conclude that for objective functions concentrated on effectiveness of

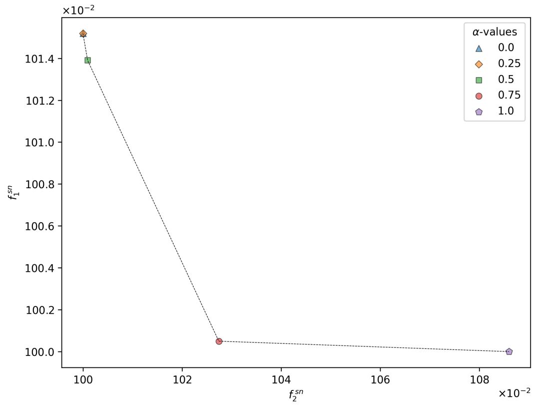


Figura 5.5: Pareto graph for instance `14n-2k-6c-8r_ML`.

PSVs' utilization, i.e., $\alpha \in \{0, 0.25\}$, the graph demonstrates f_1^{sn} approaches 1.015. This means a slight increase around of 1.5% in the fuel cost f_1, in comparison with the minimum observed. For $\alpha = 0.5$, there still is no relevant decrease in f_1^{sn}. However, if one's aim is to optimize fuel consumption, while escaping from unnecessary greater route finishing times, adopting $\alpha = 0.75$ seems a reasonable decision.

Table 5.7 presents scheduling details for the instance whose Pareto curve appears in Figure 5.5. An example on how that table should be read can described as follows. For $\alpha = 0$, the medium size PSV's availability time is $AT = 7$ h and the time planned for that vessel to start the cargo loading service at the base "b"is $s = 7$ h. After finishing the onshore service, it departures from that base and navigates to the platform number 1 to provide the service related to order 0, owned by that platform. This is represented by $(0)_1$.

Tabela 5.7: Routing and scheduling details for the instance 14n-2k-6c-8r_ML.

Instance data			Scheduling results			
α	PSV	AT	s	f	$f-s$	Route
0	`M`	7.0	7.0	100.6	93.6	b, $(0)_1, (7,8)_5, (4)_3, (1,3,2)_2, (5,6)_4, (12,13,10,11)_6$, b
	`L`	0.8	0.8	32.5	31.7	b, $(9)_5$, b
0.25	`M`	7.0	7.0	100.6	93.6	b, $(0)_1, (7,8)_5, (4)_3, (3,1,2)_2, (6,5)_4, (12,10,11,13)_6$, b
	`L`	0.8	0.8	32.5	31.7	b, $(9)_5$, b
0.5	`M`	7.0	7.3	100.7	93.4	b, $(7,8)_5, (4)_3, (1,3,2)_2, (5,6)_4, (12,10,11,13)_6, (0)_1$, b
	`L`	0.8	0.8	32.5	31.7	b, $(9)_5$, b
0.75	`M`	7.0	12.2	100.7	88.5	b, $(0)_1, (3,1,2)_2, (5,6)_4, (12,10,13,11)_6, (4)_3$, b
	`L`	0.8	0.8	35.2	34.4	b, $(9,7,8)_5$, b
1	`M`	7.0	20.1	108.6	88.5	b, $(4)_3, (6,5)_4, (10,13,12,11)_6, (3,1,2)_2, (0)_1$, b
	`L`	0.8	0.8	35.2	34.4	b, $(9,7,8)_5$, b

Afterwards, the vessel navigates to platform number 5, where it sequentially attends orders 7 and 8. These activities are represented by $(7,8)_5$. The last platform visited in the route for vessel `M` is 6, where it services orders 12, 13, 10, and 11 exactly in that sequence, returns to the base, and finishes the route at $f = 100.6$ h, yielding the total routing time $f - s = 93.6$ h. For $\alpha = 0$, a similar reading can be made for the route performed by the large PSV.

Concerning the scheduling results, the Table 5.7 demonstrates that the solutions for $\alpha \in \{0, 0.25\}$ are rather similar, presenting a few changes in the services' sequence for platforms 2, 4, and 6. Those solutions avoid idle times, since the vessels start their services right at AT, and focus on utilizing smaller PSVs. By doing this, the `M`-size PSV ends up transporting 13 out of 14 orders existing in that instance.

When α is set to the central value 0.5, the problem's objective function equally balances fuel cost and effective vessel utilization. Nevertheless, the routing times turn out to be quite similar to those for $\alpha < 0.5$, even though the `M`-size vessel's idleness increases in 0.3 h ($s - AT = 0.3$) and the sequence of platforms visited by the `M`-size vessel is rearranged so that lower routing costs are achieved.

For $\alpha = 0.75$, the importance of fuel cost is more relevant in the face of effective vessel utilization, case in which a non-intuitive, yet cost-advantageous solution is found by fully servicing the platform 5 using the large vessel, despite the fact that it consumes more fuel. In this scenario, the `M`-size PSV has its service start time at the supply base delayed in 5.2 h – i.e., the vessel is idle from 7 h to 12.2 h – since its route is now shorter. This is a better decision, since the fuel consumption for a vessel awaiting at the base vicinity is likely to be less than that in the offshore area, case in which the PSV must usually hold position.

If saving fuel is the only aspect that matters, what is achieved by $\alpha = 1$, there may be longer idle times for vessels at the base vicinity, what is observed for the `M`-size PSV, as it turns available for use at 7 h, however it begins to operate at the base only at 20.1 h, after being 13.1 h in idle state. For this scenario, the final route time $f = 108.6$ h ends up to be 8 h longer than those seen for $\alpha < 1$, what may impact forthcoming scheduling opportunities at the cost of saving bare 2% in fuel.

Although for some instances the trade-off regarding fuel cost and vessel utilization is more evident, such as that in Figure 5.5, a similar output does not hold for the average case demonstrated in Figure 5.6. This plot presents another Pareto-like curve also relating scaled, non-dimensional pairs $\overline{P}(\alpha) = \left(\overline{f}_1^{sn}(\alpha), \overline{f}_2^{sn}(\alpha)\right)$, for all $\alpha \in \mathcal{H}$, yet now as an average view. The value on each axis for the coordinates of those pairs are given by:

$$\overline{f}_1^{sn}(\alpha) = \frac{\sum_{i \in \mathcal{I}_\alpha} \left(\frac{f_1^*(i,\alpha)}{\min_{\beta \in \mathcal{H}} \left\{ f_1^*(i,\beta) \right\}} \right)}{|\mathcal{I}_\alpha|} \qquad \forall \alpha \in \mathcal{H} \tag{5.41}$$

$$\overline{f}_2^{sn}(\alpha) = \frac{\sum_{i \in \mathcal{I}_\alpha} \left(\frac{f_2^*(i,\alpha)}{\min_{\beta \in \mathcal{H}} \left\{ f_2^*(i,\beta) \right\}} \right)}{|\mathcal{I}_\alpha|} \qquad \forall \alpha \in \mathcal{H} \tag{5.42}$$

In (5.41) and (5.42), $\mathcal{I}_\alpha$ is the set of instances solved to optimality for some $\alpha \in \mathcal{H}$, and $f_1^*(i,\alpha)$ and $f_2^*(i,\alpha)$ are the optimal values for f_1 and f_2, respectively, given certain instance $i \in \mathcal{I}_\alpha$ and $\alpha \in \mathcal{H}$. Figure 5.6 reveals that the relative variability in fuel cost expressed as $\overline{f}_1^{sn}$ in that figure is negligible for all $\alpha \in \mathcal{H}$.

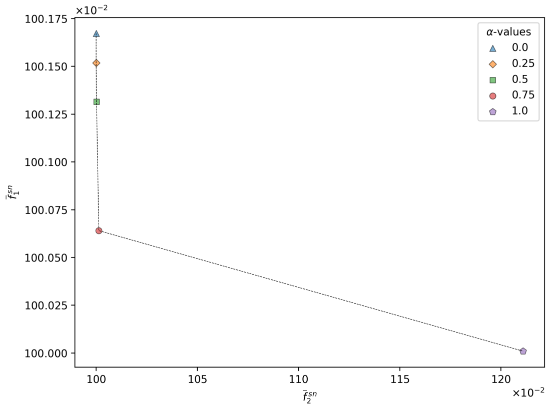


Figura 5.6: Consolidated Pareto plot.

However, for the instances studied, it can be concluded that optimizing fuel solely leads on average to utilizing vessels for longer periods, i.e., given the increase greater than 20% presented in $\overline{f}_2^{sn}$. A reasonable choice to cope with both fuel cost and vessel utilization would be $\alpha = 0.75$.

5.3 Discussion on the practical application

E&P operators and logistic services providers are frequently challenged by the task of designing good plans for the logistic system in a fast and reliable manner. Typically, such plans are created in spreadsheets using in-company, developer-specific heuristics. Despite that being ubiquitously applied, it is not uncommon for such methods to ignore important operational factors, as CIGOLINI *et al.* (2014) points out.

For this reason, designing an exact mathematical representation in the form of an MILP model is relevant to enable a structured, flexible, and easily modifiable approach to solve the d-PSVRSP. It provides generality in terms of daily usage for maritime routing and scheduling of supply vessels, while encompassing the desired operational aspects. Another frequently seen premise when designing efficient routing and scheduling plans is to assume that the transport activities have an underlying structure allowing one to regard them as a fixed, periodic tasks. There are some arguable reasons for moving forward with such a strategy.

Firstly, as general cargo is uninterruptedly demanded, it creates a perception that fixing a periodic scheme for pickup/delivery should properly suit the offshore demand as a whole. However, the cargoes are diverse in type, amount requested, and can be very installation- and/or operation-specific. For instance, rigs use to demand dry bulks and large quantities of tubular items, which are not part of a production unit's demand profile.

Secondly, it is claimed that, as production units have a long lifetime (e.g., 25–30 years) at a fixed location, there would be no decisive argument to modify the periodic routing at a higher frequency. Conversely, drilling rigs change their locations more frequently, e.g., a few weeks, hence requesting orders from different offshore locations oftentimes. Even though, given an oil company usually operates more production units than rigs, the culture of fixing the logistic plan for longer time intervals succeed.

Thirdly, logistics planners argue that there is a generalized perception of a "rhythm or cadence"in the transport activities when medium (a couple of weeks) to long (a few months) term plans are prioritized. As a consequence, this atmosphere creates an operational "comfort zone"that leverages the maintenance of fixed, periodic logistic plans for the longest time possible.

Regardless the overall result that this approach produces, installations and logistics personnel look at it positively, as it contributes to the sense of a controllable, predictable service performance over time. This course of action has motivated several publications on operational planning of PSVs to develop their models as *periodic vehicle routing problems* (PVRPs). A few examples of papers that regard the problem in this manner are FAGERHOLT (2000), HALVORSEN-WEARE and FAGERHOLT (2011), HALVORSEN-WEARE *et al.* (2012b), KISIALIOU *et al.* (2018b), and CRUZ *et al.* (2019).

Given the existence of steady and fluctuating demands, different types of platforms, heterogeneous PSVs, specific time constraints (e.g., time windows, deadlines), and the expectation for adherence to what is planned, the ability to generate good quality routing and scheduling plans in a relatively short time is critical, since in the operational level there will usually be few hours to make and/or correct transport plans. In this sense, the results presented indicate that the method to solve the d-PSVRSP can be used in practice for groups that demand up 14 orders, since beyond that, i.e., 16- to 20-order size problems, the method's performance decreases.

At last, the possibility to schedule more than one trip per vessel is an interesting feature for use mainly when one faces restricted transportation capacity at the target planning horizon. This situation is not rare in practice, since adverse offshore environmental conditions prolong the trip time significantly, reducing the number of vessels available for use at the supply base's vicinity area. Finally, the MILP model is general enough to decide for specialization, e.g., a route that delivers only diesel, or to find solutions in which a PSV carries multiple commodity types, simultaneously.

Capítulo 6

Routing and scheduling of platform supply vessels under uncertainty

The previous chapter assessed the benefits for practical use of a mathematical programming model for routing and scheduling of PSVs relying on average-valued parameters. This approach is important for constituting an ideal referential for an offshore service plan and cost estimates assumed free of external, uncontrollable uncertainties, such as adverse environmental conditions. Matter of fact, in many situations the routes one plans do not suffer any damage from exogenous uncertainty simply because favorable offshore scenarios unfold, in which for instance only minor or no delays at all occur.

However, it is desirable that routing plans be able to cope with uncertainty, whenever it realizes. In order to consider it in an optimization schema, it may seem reasonable to solve an optimization model separately for the realizations of the random parameters, obtain optimal pairs of objective function and solution, and somehow combine them all as single objective and solution values, hoping that this approach provides some protection against uncertainty. The drawback of doing this is the numerous optimization problems that must be solved, as the number of scenarios increase extremely fast with the problem size. Another issue is the “somehow", as it may not be simple to combine all solutions suitably.

Other natural temptation is to conduct sensitivity analysis with the problem random parameters – honestly understood as random, yet maybe modeled as single-averaged values to ease the solution process – aiming to “tune"the optimization model to better face uncertainty. HIGLE (2005) discourages this idea, arguing that in many cases sensitivity analysis leads to a false sense of security against uncertainty, and, if uncertainty data is available for random parameters, the optimization model should embrace it.

Usually, stochastic optimization models consider decision stages, since the realization of uncertainty data can occur in distinct time moments. The general formulation of a linear two-stage SPR, one of the most seen types of stochastic programs, is given by:

$$
\begin{aligned}
z = \operatorname*{Min}_{x}\ & c^{\top}x + \mathbb{E}_{\xi}\left[Q(x,\xi(\omega))\right] \\
s.t.\ & Ax = b \\
& x \geqslant 0,
\end{aligned}
\tag{6.1}
$$

where

$$
\begin{aligned}
Q(x,\xi(\omega)) = \operatorname*{Min}_{y}\ & q(\omega)^{\top}y(\omega) \\
s.t.\ & W(\omega)y(\omega) = h(\omega) - T(\omega)x \\
& y(\omega) \geqslant 0.
\end{aligned}
\tag{6.2}
$$

In 6.1, c denotes the first-stage cost vector; x denotes the first-stage decision vector; ξ is a random variable representing uncertainty; ω is an outcome of ξ usually called *scenario*, belonging to the set of all scenarios, denoted by Ω; $\mathbb{E}$ denotes expectation; $Ax = b$ defines the first-stage constraints. In 6.2, y denotes the second-stage decision vector; $\xi(\omega) = (q(\omega), h(\omega), T(\omega))$ are second-stage parameters dependent on the scenario ω; and $W(\omega)y(\omega) = h(\omega) - T(\omega)$ defines the second-stage constraints. The term $Q(x,\xi(\omega))$ corresponds to the optimal value of the second-stage recourse problem given x and a realization $\xi(\omega), \omega \in \Omega$.

It is usual to define the expected second-stage value function as:

$$
\mathcal{L}(x) = \mathbb{E}_{\xi}\left[Q(x,\xi(\omega))\right], \tag{6.3}
$$

which allows one to represent 6.1 in its full form, usually referred to as *deterministic equivalent problem* (DEP), defined as follows (BIRGE and LOUVEAUX, 2011).

$$
\begin{aligned}
z_{\mathrm{DEP}} = \operatorname*{Min}_{x}\ & c^{\top}x + \mathcal{L}(x) \\
s.t.\ & Ax = b \\
& x \geqslant 0.
\end{aligned}
\tag{6.4}
$$

The term $\mathcal{L}(x)$ is the main difference for a simple deterministic formulation. The domain of x in 6.4 or y in 6.2 can be suitably adjusted so that the problem could be turned into a discrete one, for instance. When $\mathcal{L}(x)$ is tractable ("computable"), it can be accordingly replaced by relations that can be formulated as a DEP, which in turn can solved with traditional methods for MILP models.

Given this context, this chapter introduces a mathematical formulation of an MILP model in the form of a DEP to solve the s-PSVRSP. Besides, it also presents a series of experimental results obtained from solving artificial instances inspired from real life data of an oil and gas operator that develops its offshore logistics activities in southeast Brazilian waters, and a discussion on the practical application of the method proposed.

6.1 Mathematical modeling

This section presents an MILP formulation for the s-PSVRSP. Before introducing such a formulation, it is worth to mention a few aspects of the modeling approach, in order to facilitate the understanding the MILP model by the reader. Firstly, the formulation does not regard the problem as a periodic one in the sense of fixing *a priori* the number of visits that a platform can have within a pre-specified time horizon or even within a trip. The periodic aspect in the present problem relies only on the fact that each cluster's platforms place cargo delivery and pickup orders daily, which in turn are attended from a fixed, regular, and previously defined scheme of PSV daily departures from the supply base.

Secondly, the formulation proposed actually routes PSVs to "visit"cargo orders, which means that when a vessel arrives at a platform, it does not necessarily have to fulfill all orders requested by such a platform at a single visit. As a platform and the orders placed by it share the same location coordinates, the sequence of platforms visited implicitly results from the routing solutions found with the MILP model for the s-PSVRSP. Figure 1.2b illustrates a few routing examples that consider orders as visit nodes, instead of platforms themselves. At last, the formulation also allows multiple visits at a platform per trip, with a same vessel or with a different one. This can achieved due to the fact that the nodes visited by a PSV are actually orders. Before presenting the MILP formulation, additional notation is introduced for convenience.

6.1.1 Specific notation

This section presents the notation necessary specifically for the s-PSVRSP.

Sets

$\mathcal{C}$	Set of maritime platforms, defined as: $\mathcal{C} := \{1, 2, \ldots, b\}$.
$\mathcal{P}$	Set of commodity types.
$\mathcal{O}$	Set of orders requested by maritime platforms, defined as: $\mathcal{O} := \{1, 2, \ldots, n\}$.
$\mathcal{W}_i$	Set of time windows for order $i \in \mathcal{O}$ requested by platform $c^i \in \mathcal{C}$.
$\mathcal{V}$	Set of PSVs.
$\mathcal{Q}$	Set of compartment types for a vessel.

$\mathcal{P}_q \subseteq \mathcal{P}$	Set of commodity types compatible with compartment type $q \in \mathcal{Q}$.
Ω	Set of scenarios.
$\mathcal{N}$	Set of nodes, defined as $\mathcal{N} \coloneqq \mathcal{O} \cup \{0, n+1\}$, in which define 0 and $n+1$ are initial and final supply base copies, respectively, for modeling purposes.
$\mathcal{A}$	Set of arcs, defined as: $\mathcal{A} \coloneqq \{(i,j) : (i,j) \in \mathcal{N} \times \mathcal{N}, i \neq j, i \neq n+1, j \neq 0\}$.
$\mathcal{N}_-$	Set of nodes, except node $n+1$, defined as: $\mathcal{N}_- \coloneqq \mathcal{N} \setminus \{n+1\}$.
$\mathcal{N}_+$	Set of nodes, except node 0, defined as: $\mathcal{N}_+ \coloneqq \mathcal{N} \setminus \{0\}$.
$\mathcal{A}_o$	Set of arcs for order pairs, defined as: $\mathcal{A}_o \coloneqq \{(i,j) : (i,j) \in \mathcal{O} \times \mathcal{O}, i \neq j\}$.
$\hat{\mathcal{A}}$	Set of arcs for pairs of orders to prevent overlapping services, defined as: $\hat{\mathcal{A}} \coloneqq \{(i,j) : (i,j) \in \mathcal{A}_o, i < j, c^i \equiv c^j, c^i, c^j \in \mathcal{C}\}$.
$\mathcal{A}_-$	Set of arcs from supply base node 0 to an order, defined as: $\mathcal{A}_- \coloneqq \{(i,j) : i \equiv 0, j \in \mathcal{O}\}$.
$\mathcal{A}_+$	Set of arcs from an order to supply base node $n+1$, defined as: $\mathcal{A}_+ \coloneqq \{(i,j) : i \in \mathcal{O}, j \equiv n+1\}$.
$\mathcal{A}_\equiv$	Set of arcs between orders that belong to the same platform, defined as: $\mathcal{A}_\equiv \coloneqq \{(i,j) : (i,j) \in \mathcal{A}_o, c^i \equiv c^j, c^i, c^j \in \mathcal{C}\}$.
$\mathcal{A}_{\neq}$	Set of arcs between orders that belong to different platforms, defined as: $\mathcal{A}_{\neq} \coloneqq \{(i,j) : (i,j) \in \mathcal{A}_o, c^i \neq c^j, c^i, c^j \in \mathcal{C}\}$.
$\overline{\mathcal{A}}$	Set of arcs, except those for order pairs of different platforms, the defined as: $\overline{\mathcal{A}} \coloneqq \mathcal{A} \setminus \mathcal{A}_{\neq}$.
$\underline{\mathcal{A}}$	Set of arcs, except those for order pairs of the same platform, defined as: $\underline{\mathcal{A}} \coloneqq \mathcal{A} \setminus \mathcal{A}_\equiv$.

Parameters

ϕ_p	Loading/unloading rate of commodity $p \in \mathcal{P}$ at a platform.
σ_p	Loading/unloading rate of commodity $p \in \mathcal{P}$ at the supply base.
B_{ip}	Pickup quantity of commodity $p \in \mathcal{P}$, for order $i \in \mathcal{O}$.
L_{ip}	Delivery quantity of commodity $p \in \mathcal{P}$, for order $i \in \mathcal{O}$.
ET_{ih}	Earliest time point for time window $h \in \mathcal{W}_i$ for order $i \in \mathcal{O}$.
LT_{ih}	Latest time point for time window $h \in \mathcal{W}_i$ for order $i \in \mathcal{O}$.
ζ_i	Penalty per hour for violating ET_{ih}, $i \in \mathcal{O}$, $h \in \mathcal{W}_i$.
β_i	Penalty per hour for violating LT_{ih}, $i \in \mathcal{O}$, $h \in \mathcal{W}_i$, defined as: $\beta_i = 2\zeta_i$.
Q_q^k	Capacity of compartment type $q \in \mathcal{Q}$ for vessel $k \in \mathcal{V}$.
AT^k	Moment at which vessel $k \in \mathcal{V}$ becomes available for loading at the base.
N_{ij}^k	Navigation time for vessel $k \in \mathcal{V}$ from c^i to c^j, for $c^i, c^j \in \mathcal{C}$, $i, j \in \mathcal{O} \cup \{0\}$, $i \neq j$.
SE^k	Setup time for vessel $k \in \mathcal{V}$ before it departures from a platform.
SP^k	Safe positioning time for vessel $k \in \mathcal{V}$ when arriving at a platform.

T_{ij}^k Total travel time for vessel $k \in \mathcal{V}$ from c^i to c^j, $c^i, c^j \in \mathcal{C}$, $i, j \in \mathcal{O} \cup \{0\}, i \neq j$, defined in 3.2.

L^k Maximum number of consecutive trips allowed for vessel $k \in \mathcal{V}$.

TD^k Maximum trip duration for vessel $k \in \mathcal{V}$.

φ^k Fuel cost per hour for $k \in \mathcal{V}$ operating at the supply base.

γ^k Fuel cost per hour for $k \in \mathcal{V}$ in navigation.

δ^k Fuel cost per hour for $k \in \mathcal{V}$ enrolled in service-related tasks offshore, such as cargo handling, waiting times, or setups.

π_ω Probability of scenario $\omega \in \Omega$, $\pi \in [0, 1]$.

S_{iw} Delay that a vessel incurs before serving order $i \in \mathcal{O} \cup \{0\}$ at scenario $\omega \in \Omega$.

6.1.2 Modeling

The constraints and objective function for the s-PSVRSP are defined as follows. The constraints and objective function of such a formulation are introduced in “blocks". The first block presents the degree constraints, which produce the route's structure. The second block accounts for the commodity flow constraints. The third block of constraints accounts for the time windows to be used. The fourth block of constraints serves the purpose of avoiding service times to overlap for each platform's orders. The fifth block defines constraints that express time moments. The sixth block of constraints defines the first and second stage costs considered in the problem. At last, the problem's objective function is presented.

Nodes' degree constraints. Let a binary variable x_{ij}^k be 1 if and only if vessel $k \in \mathcal{V}$ traverses arc $(i, j) \in \mathcal{A}$. Constraints (6.5) and (6.6) enforce that every vessel in use must start its route at the supply base and return back to it after finishing its offshore schedule. Case a vessel is not used, it fictitiously travels from node 0 directly to $n + 1$. Constraints (6.7) state each order's degree must be two, i.e., a vessel arrives to serve it, and departures afterwards. Constraints (6.8) assure that every order will be served exactly once.

$$\sum_{j \in \mathcal{N}_+} x_{0,j}^k = 1 \qquad \forall k \in \mathcal{V} \tag{6.5}$$

$$\sum_{i \in \mathcal{N}_-} x_{i,n+1}^k = 1 \qquad \forall k \in \mathcal{V} \tag{6.6}$$

$$\sum_{i \in \mathcal{N}_-} x_{io}^k = \sum_{j \in \mathcal{N}_+} x_{oj}^k \qquad \forall k \in \mathcal{V}, \forall o \in \mathcal{O} \tag{6.7}$$

$$\sum_{j \in \mathcal{N}_+} \sum_{k \in \mathcal{V}} x_{ij}^k = 1 \qquad \forall i \in \mathcal{O} \tag{6.8}$$

Commodity flow constraints. Let a non-negative variable z_{ip}^k be an amount of the commodity $p \in \mathcal{P}$ carried by a vessel $k \in \mathcal{V}$ when it visits node $i \in \mathcal{N}$. Constraints (6.9) capture the quantity of each commodity loaded into a vessel before it departures from the supply base. Constraints (6.10) assure each vessel's capacity is respected. Constraints (6.11) and (6.12), which also avoid formation of sub-tours, correspond to the linearization of the constraints

$$x_{ij}^k \left(z_{ip}^k + B_{jp} - L_{jp} - z_{jp}^k\right) = 0 \qquad \forall k \in \mathcal{V}, \forall (i,j) \in \mathcal{A}, \forall p \in \mathcal{P},$$

which account for the net commodity quantity carried by a vessel at each node that it visits. In this linearization, M_1 is a large enough number that turns constraints (6.11) and (6.12) not binding case x_{ij}^k assumes value 0 for some $k \in \mathcal{V}$ and $(i,j) \in \mathcal{A}$.

$$\sum_{i \in \mathcal{O}} \sum_{j \in \mathcal{N}_+} L_{ip} x_{ij}^k = z_{0,p}^k \qquad \forall k \in \mathcal{V}, \forall p \in \mathcal{P} \tag{6.9}$$

$$\sum_{p \in \mathcal{P}_q} z_{ip}^k \leqslant Q_q^k \left(1 - x_{0,n+1}^k\right) \qquad \forall k \in \mathcal{V}, \forall i \in \mathcal{N}, \forall q \in \mathcal{Q} \tag{6.10}$$

$$z_{jp}^k \leqslant z_{ip}^k + B_{jp} - L_{jp} + M_1 \left(1 - x_{ij}^k\right) \qquad \forall k \in \mathcal{V}, \forall (i,j) \in \mathcal{A}, \forall p \in \mathcal{P} \tag{6.11}$$

$$z_{jp}^k \geqslant z_{ip}^k + B_{jp} - L_{jp} - M_1 \left(1 - x_{ij}^k\right) \qquad \forall k \in \mathcal{V}, \forall (i,j) \in \mathcal{A}, \forall p \in \mathcal{P} \tag{6.12}$$

Soft time window constraints. Given a vessel $k \in \mathcal{V}$ and a scenario $\omega \in \Omega$, let a binary variable r_i^k be 1 if and only if k visits order $i \in \mathcal{O}$; binary variable u_{ih}^k be 1 if and only if k performs the service related to i within the window $h \in \mathcal{W}_i$; non-negative variables $d_{i\omega}^k$ and $a_{j\omega}^k$, which are the time that k departures from $i \in \mathcal{N}_-$, and the time that such a vessel arrives at $j \in \mathcal{N}_+$, respectively; non-negative variable $e_{i\omega}^k$, which is the violation extent that k incurs when it starts the service at i before ET_{ih}; and non-negative variable $l_{i\omega}^k$, which is the violation extent that k incurs when it finishes the service at i after LT_{ih}. Constraints (6.13) enforce that exactly one time window must be chosen, whereas they also indicate what vessel fulfills what order. Together, constraints (6.14) and (6.15) induce services to happen within time windows, yet allow them to violate such windows.

$$r_i^k = \sum_{h \in \mathcal{W}_i} u_{ih}^k \qquad \forall k \in \mathcal{V}, \forall i \in \mathcal{O} \tag{6.13}$$

$$a_{i\omega}^k + e_{i\omega}^k \geqslant \sum_{h \in \mathcal{W}_i} ET_{ih} u_{ih}^k \qquad \forall \omega \in \Omega, \forall k \in \mathcal{V}, \forall i \in \mathcal{O} \tag{6.14}$$

$$d_{i\omega}^k - l_{i\omega}^k \leqslant \sum_{h \in \mathcal{W}_i} LT_{ih} u_{ih}^k \qquad \forall \omega \in \Omega, \forall k \in \mathcal{V}, \forall i \in \mathcal{O} \tag{6.15}$$

Non-overlapping service constraints. Let a binary variable m_{ij} be 1 if and only if the service related to order i finishes before that one associated with order j, given $(i,j) \in \hat{\mathcal{A}}$. Case $|\mathcal{V}| > 1$, constraints (6.16) and (6.17) must be included in the model to avoid service overlap among orders of a same platform. Case $|\mathcal{V}| = 1$, there will be no overlapping, since the existing PSV will fulfill all orders in the route. Let M_2 be a large enough number that turns either (6.16) or (6.17) not binding, depending on what order, i or j, is visited first.

$$d_{jw}^k \leqslant a_{iw}^k + M_2 m_{ij} \qquad \forall \omega \in \Omega, \forall k \in \mathcal{V}, \forall (i,j) \in \hat{\mathcal{A}} \tag{6.16}$$

$$d_{iw}^k \leqslant a_{jw}^k + M_2 (1 - m_{ij}) \qquad \forall \omega \in \Omega, \forall k \in \mathcal{V}, \forall (i,j) \in \hat{\mathcal{A}} \tag{6.17}$$

Time constraints. Given a vessel $k \in \mathcal{V}$, a node $j \in \mathcal{N}_+$, and a scenario $\omega \in \Omega$, let a non-negative variable s_0^k be the time that k starts to load at the supply base; non-negative variable $t_{j\omega}^k$ be the waiting time that k spends before starting service at j; and a non-negative variable $f_{n+1,\omega}^k$ be the time that k finishes to unload at the base. Constraints (6.18) and (6.19) define the supply base loading and unloading times, respectively. Constraints (6.20) establish each order's service duration, whereas constraints (6.21) and (6.22) correspond to the linearization of the constraints

$$x_{ij}^k \left(d_{i\omega}^k + T_{ij}^k + t_{j\omega}^k + S_{j\omega} - a_{j\omega}^k \right) = 0 \qquad \forall \omega \in \Omega, \forall k \in \mathcal{V}, \forall (i,j) \in \mathcal{A},$$

which account for the time elapsed when a vessel traverses the arc (i,j). The constant M_3 is a large enough number that turns constraints (6.21) and (6.22) not binding case x_{ij}^k assumes value 0 for some $k \in \mathcal{V}$ and $(i,j) \in \mathcal{A}$.

$$s_0^k + \sum_{p \in \mathcal{P}} \sigma_p z_{0,p}^k = d_{0,\omega}^k \qquad \forall \omega \in \Omega, \forall k \in \mathcal{V} \tag{6.18}$$

$$a_{n+1,\omega}^k + \sum_{p \in \mathcal{P}} \sigma_p z_{n+1,p}^k = f_{n+1,\omega}^k \qquad \forall \omega \in \Omega, \forall k \in \mathcal{V} \tag{6.19}$$

$$a_{i\omega}^k + \sum_{p \in \mathcal{P}} (L_{ip} + B_{ip}) \phi_p r_i^k = d_{i\omega}^k \qquad \forall \omega \in \Omega, \forall k \in \mathcal{V}, \forall i \in \mathcal{O} \tag{6.20}$$

$$a_{j\omega}^k \leqslant d_{i\omega}^k + T_{ij}^k + t_{j\omega}^k + S_{j\omega} + M_3 \left(1 - x_{ij}^k\right) \qquad \forall \omega \in \Omega, \forall k \in \mathcal{V}, \forall (i,j) \in \mathcal{A} \tag{6.21}$$

$$a_{j\omega}^k \geqslant d_{i\omega}^k + T_{ij}^k + t_{j\omega}^k + S_{j\omega} - M_3 \left(1 - x_{ij}^k\right) \qquad \forall \omega \in \Omega, \forall k \in \mathcal{V}, \forall (i,j) \in \mathcal{A} \tag{6.22}$$

First and second-stage cost constraints. Given a vessel $k \in \mathcal{V}$, let the non-negative variables v^k and y_ω^k, $\omega \in \Omega$, represent the first and second-stage costs, respectively. Constraints (6.23) correspond to the deterministic fuel costs per vessel associated with using it at the supply base, in navigation, and in performing the offshore orders' service, setups, and safe positioning occurrences. In these constraints, the cost C_{ij}^k, $(i,j) \in \mathcal{A}$ is defined as:

$$C_{ij}^k := \begin{cases} \gamma^k N_{ij}^k + \delta^k \left[SP^k + \sum_{p \in \mathcal{P}} \phi_p \left(L_{jp} + B_{jp} \right) \right], & \text{if } (i,j) \in \mathcal{A}_- \\ \gamma^k N_{ij}^k + \delta^k \left[SE^k + SP^k + \sum_{p \in \mathcal{P}} \phi_p \left(L_{jp} + B_{jp} \right) \right], & \text{if } (i,j) \in \mathcal{A}_{\neq} \\ \delta^k \sum_{p \in \mathcal{P}} \phi_p \left(L_{jp} + B_{jp} \right), & \text{if } (i,j) \in \mathcal{A}_{\equiv} \\ \delta^k SE^k + \gamma^k N_{ij} & \text{if } (i,j) \in \mathcal{A}_+ \\ 0, & \text{if } (i,j) \equiv (0, n+1) \end{cases}$$

Constraints (6.24) represent the offshore fuel costs per vessel that arise from the combination of waiting, realizations of $S_{i\omega}$, $i \in \mathcal{O}$, $\omega \in \Omega$, and time window violations. Parameters ζ_i and β_i, $i \in \mathcal{O}$ denote penalties for violating a time window. Variable y_ω^k is regarded as a consolidated recourse action that integrates corrective decisions associated with waiting and violations. For any random event, i.e., offshore environmental specific conditions that lead to $S_{i\omega} > 0$, the second-stage will observe at least an inevitable fuel cost given by $\delta^k S_{i\omega} r_i^k$ if $r_i^k = 1$, for some $k \in \mathcal{V}$, $i \in \mathcal{O}$. In this situation, no recourse action will be capable of mitigating such a cost. It will be just absorbed as additional fuel cost due to awaiting for favorable operational conditions.

$$v^k = \sum_{p \in \mathcal{P}} \varphi^k \sigma_p \left(z_{0,p}^k + z_{n+1,p}^k \right) + \sum_{(i,j) \in \mathcal{A}} C_{ij}^k x_{ij}^k \qquad \forall k \in \mathcal{V} \tag{6.23}$$

$$y_\omega^k = \sum_{i \in \mathcal{O}} \delta^k \left(t_{i\omega}^k + S_{iw} r_i^k \right) + \zeta_i e_{i\omega}^k + \beta_i l_{i\omega}^k \qquad \forall \omega \in \Omega, \forall k \in \mathcal{V} \tag{6.24}$$

Objective function. The problem's objective is presented in (6.25). The summations $\sum_{k \in \mathcal{V}} v^k$ and $\sum_{\omega \in \Omega} \sum_{k \in \mathcal{V}} \pi_\omega y_\omega^k$ express operational fleet costs, the former related to first-stage routing decisions and, the latter, an expected cost associated with second-stage scheduling decisions regarded as recourse actions, given scenario probabilities π_ω, $\omega \in \Omega$.

$$\underset{v,y}{\text{Min}} \sum_{k \in \mathcal{V}} v^k + \sum_{k \in \mathcal{V}} \sum_{\omega \in \Omega} \pi_\omega y_\omega^k \tag{6.25}$$

The goal of the s-PSVRSP is to perform the minimization expressed in (6.25) subject to constraints (6.5) – (6.24).

6.1.3 Sample average approximation (SAA)

Solving a deterministic equivalent program such as the s-PSVRSP, expressed in 6.5 – 6.25, is particularly difficult for the large number of implicit second-stage optimization problems $Q(x, \xi(\omega))$ that need to be solved. Another aspect that increases the problem's complexity is the number of scenarios $|\Omega|$, which usually escalates quickly in real applications, leading to a huge number of second-stage constraints and variables, as they are all ω-indexed.

As an example, suppose a routing problem with 10 service nodes (e.g., orders, clients), each of these nodes having a random parameter with 20 discrete values to describe the realization of delays $S_{i\omega}, i \in \mathcal{O}, \omega \in \Omega$. This would result $|\Omega| = 10^{20}$ scenarios, leading to an extremely large model, for sure unsolvable in an acceptable time, therefore of no value for the practical purpose of finding high quality routing solutions.

An alternative to mitigate this sort of problem is applying the *sample average approximation* (SAA) method, which provides more tractability to stochastic optimization problems, such as the s-PSVRSP, for approximating the implicit calculation of the expectation $\mathbb{E}_\xi \left[Q(x, \xi(\omega))\right]$ existing in the model, by the sample average function

$$\frac{1}{N}\sum_{n=1}^{N} Q(x, \xi(\omega^n),$$

which uses a countable number of scenarios N. By generating N samples $\omega^1, \omega^2, \ldots, \omega^N$ from Ω according to the a sound probability distribution – a distribution for the random parameter S in the case of the s-PSVRSP – it is possible to solve a deterministic optimization problem employing the sample average function and such samples. This deterministic optimization problem for the SAA method can be expressed as:

$$z_N = \underset{x \in X}{\text{Min}}\ c^\top x + \frac{1}{N}\sum_{n=1}^{N} Q(x, \xi(\omega^n)), \tag{6.26}$$

and corresponds to the original, so-called “true"problem, formulated in terms of an expectation as second-stage cost, being given by:

$$z^* = \underset{x \in X}{\text{Min}}\ c^\top x + \mathbb{E}_\xi \left[Q(x, \xi(\omega))\right], \tag{6.27}$$

where X denotes the first-stage feasible set of decisions given the first-stage constraints. The optimal value z_N and its optimal solution $\hat{x}$ for the SAA problem in 6.26 yield estimates z^* and x^* of the true problem in 6.27.

The general SAA method steps, further detailed in VERWEIJ *et al.* (2003), applied to the MILP minimization problem in 6.5 – 6.25 can be described as follows.

1. **Lower bound estimation**: generate M independent scenario samples of size N from Ω using a known distribution, then solve M SAA problems based on 6.26. This results in a series of objective values $z_N^1, z_N^2, \ldots, z_N^M$, as well as their associated solution estimates $\hat{x}^1, \hat{x}^2, \ldots, \hat{x}^M$, called solution candidates. This first phase of the method finishes with the calculation of an average value of the M SAA problems solved, which is represented by:

$$\bar{z}_N = \frac{1}{M}\sum_{m=1}^{M} z_N^m. \tag{6.28}$$

   According to VERWEIJ *et al.*, $\bar{z}_N$ is actually an estimate for a lower bound on the optimal value z^* in 6.27, i.e., $\mathbb{E}\left[\bar{z}_N\right] \leqslant z^*$. The standard deviation of the estimator $\bar{z}_N$ can be calculated as: The variance of the estimator $\bar{z}_N$ can be calculated as:

$$\hat{\sigma}^2_{\bar{z}_N} = \frac{1}{M(M-1)}\sum_{m=1}^{M}\left(z_N^m - \bar{z}_N\right)^2. \tag{6.29}$$

2. **Upper bound estimation**: given a first-stage feasible decision $\hat{x} \in X$, such as any of those candidates, an upper bound for z^* can be estimated by computing the following quantity:

$$\hat{z}_{N'}(\hat{x}) = c^\top \hat{x} + \frac{1}{N'}\sum_{n=1}^{N'} Q(\hat{x}, \xi(\omega^n)), \tag{6.30}$$

   where $\left\{\omega^1, \omega^2, \ldots, \omega^{N'}\right\}$ is a sample of size N' that is commonly large, respects $N' > N$, and preserves independence of a sample that might have been used to generate $\hat{x}$. Assuming these conditions hold, $\hat{z}_{N'}(\hat{x})$ can be regarded as an unbiased estimator of $c^\top \hat{x} + \mathbb{E}_\xi\left[Q(\hat{x}, \xi(\omega))\right]$, which, again according to VERWEIJ *et al.*, results in $\mathbb{E}\left[\hat{z}_{N'}(\hat{x})\right] \geqslant z^*$ holding for any feasible solution $\hat{x}$. The variance of the estimator $\hat{z}_{N'}(\hat{x})$ can be calculated as:

$$\hat{\sigma}^2_{\hat{z}_{N'}(\hat{x})} = \frac{1}{N'(N'-1)}\sum_{n=1}^{N'}\left(c^\top \hat{x} + Q(\hat{x}, \xi(\omega^n)) - \hat{z}_{N'}(\hat{x})\right)^2. \tag{6.31}$$

3. **Solution selection**: a solution can be estimated from the M candidates by taking the one that yields the best objective value. This can be achieved with

the following verification:

$$\hat{x}^* \in \arg\min \left\{ \hat{z}_{N'}(\hat{x}) : \hat{x} \in \left\{ \hat{x}^1, \hat{x}^2, \ldots, \hat{x}^M \right\} \right\}. \tag{6.32}$$

4. **Solution quality assessment**: given the lower and upper bound estimates, the quality of the solution $\hat{x}^*$ obtained can be assessed by computing its absolute optimality gap estimate

$$\hat{z}_{N'}(\hat{x}) - \bar{z}_N, \tag{6.33}$$

and associated variance:

$$\hat{\sigma}^2_{\hat{z}_{N'}(\hat{x}) - \bar{z}_N} = \hat{\sigma}^2_{\hat{z}_{N'}(\hat{x})} + \hat{\sigma}^2_{\bar{z}_N}. \tag{6.34}$$

6.1.4 Value of the stochastic solution (VSS)

After solving a stochastic optimization model, it is important to quantify how beneficial was in practice the decision of introducing uncertainty in the model. The procedure adopted for the d-PSVRSP is the *value of the stochastic solution* (VSS), as described in BIRGE and LOUVEAUX (2011). The VSS measures how much is lost for not considering uncertainty in the model. The next steps present how to compute the VSS, according to BIRGE and LOUVEAUX.

1. **Computation of the expected value problem**: the *expected value problem* (EV) is computed by: (i) replacing the stochastic parameter ξ in use in 6.1 by its average value, $\bar{\xi} = \mathbb{E}_{\xi}[\xi]$; and (ii) solving the new single-scenario (the "average scenario") deterministic optimization problem to obtain the optimal first-stage decision x_{EV} and the optimal objective value:

$$z_{\text{EV}} = z(x, \bar{\xi}). \tag{6.35}$$

2. **Impact of using the expected value problem**: this impact is known as the *expected result of using the expected value problem* (EEV). The EEV is computed as follows: (i) fix in 6.1 the first-stage decision x_{EV} obtained from the EV; and (ii) solve it, allowing the model to make the best possible second-stage decision y given the fixation of x_{EV}. This procedure allows to evaluate how x_{EV} performs when uncertainty takes place, i.e., when scenarios realize. The objective value for the EEV is given by:

$$z_{\text{EEV}} = z(x_{\text{EV}}, \xi). \tag{6.36}$$

3. **Definition of value of the stochastic solution**: the *value of the stochastic solution* (VSS) is the cost of leaving the uncertainty apart when making a decision. The problem in 6.1 is also referred to as *recourse problem* (RP). Defining $z_{\mathrm{RP}} = z(x, \xi)$, the value of the stochastic solution is then as:

$$VSS = z_{\mathrm{EEV}} - z_{\mathrm{RP}}. \tag{6.37}$$

6.2 Computational studies

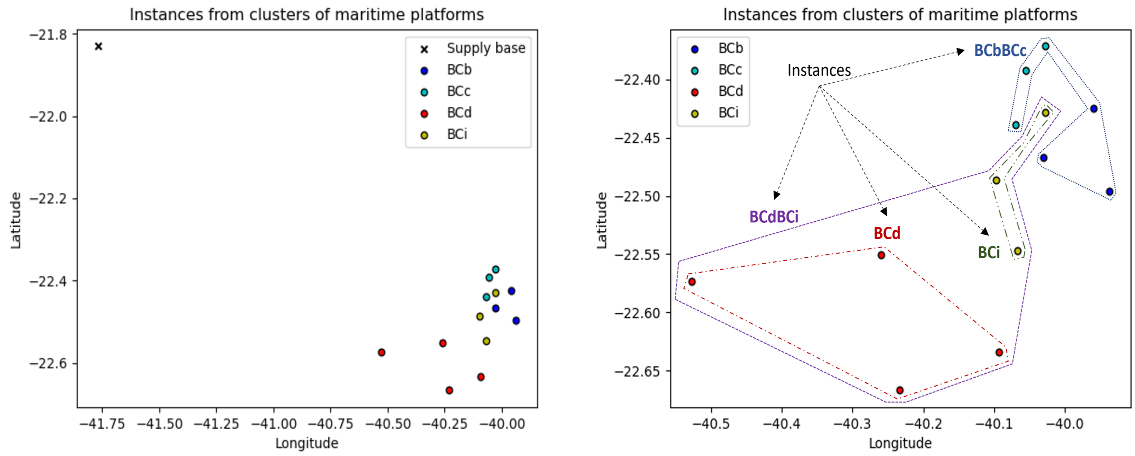
In this section the instances generation process is described, as well as the computational results obtained from applying the method proposed to the s-SPSVRSP. Implementations are made in Python 3.10.6 and all subordinate linear and mixed-integer linear programs are solved using the Gurobi Optimizer 10.0.0 through the Python application programming interface, with all settings default, except the following: `MIPGap = 0.5%` (relative gap tolerance), `TimeLimit = 3600` seconds, `NoRelHeurWork = 7`, `MIPFocus = 3`, `Cuts = 1`, and `Threads = 12`.

The gap value reported by the solver is defined as: $gap = \frac{UB-LB}{UB} \times 100$, in which UB stands for "upper bound"and LB stands for "lower bound". These parameter values were defined empirically from some tests made with a few instances, from which solutions were obtained faster than in the case that only the solver's default parameters were set. The `NoRelHeurWork` parameter intensifies the heuristic solver's search for solutions before the internal solver's solution process starts, which leads one to obtaining primal solutions of good quality. The `MIPFocus` parameter intensifies the solver's effort to improve the lower bound of the problem. The `Cuts` parameter promotes a moderate generation of cutting planes to speed up the solution process. The `Threads` parameter limits the number of processing threads offered to the solver. Higher values for this parameter deliver more processing power for the solver to manage along its solution process. The reader is referred to the solver's website GUROBI (2023) for further details about these parameters.

All computations were performed on an Intel Xeon CPU W-10885M running at 2.40 GHz. A total of 32GB of available RAM were dedicated to instances run serially (once at a time) using at most 12 threads. All CPU times and relative optimality gap values presented as results in this section are calculated as arithmetic means.

6.2.1 Benchmark instances

The instance test set for the s-PSVRSP contains 4 basic feasible instances designed as follows. There are 13 platforms that were used in the design of benchmark instances. The location of the maritime platforms are illustrated in Figure 6.1a and the maritime platforms related to the 4 instances appear in Figure 6.1b. The descriptive statistics of these platforms are similar to those described in section 5.2.1.



(a) Operator's clusters. (b) Studies' instances†.

Figura 6.1: Maritime platforms, clusters, and instances representation. †Supply base omitted to ease reader's visualization.

There are 3 sorts of commodities, general cargo, diesel, and water, whose orders and quantity per order are devised per platform from the operator's historical data. The amount of such commodities per order is also typical. Each of these commodity types has its own onshore and offshore handling efficiency given in h/m^2 for deck cargo and h/m^3 for diesel and water. Pickup and delivery orders of deck cargo are regarded a single order, whereas diesel and water appear only as deliveries, such as seen in practice.

So, for example, if a platform P_1 requests 100 m^2 of deck cargo to be delivery, 70 m^2 of deck cargo to be picked up, and 600 m^3 of diesel, and another platform P_2 requests 120 m^2 of deck cargo to be delivery, 80 m^2 of deck cargo to be picked up, and 400 m^3 of water, P_1 and P_2 will both be regarded as having 3 orders each, being each of these orders associated with the coordinates of the platform they belong to. In other words, for P_1 there will be one order for deck cargo (delivery and pickup together), one for diesel, and another one for water, whereas for P_2 there will be 3 orders as well, but the last one will be water. Case P_1 and P_2 were the only platforms of a cluster, it would have 6 orders (service nodes) to be fulfilled in a PSV's route.

Time windows, in the context of the s-PSVRSP, take place due to solely the existence of some platforms that are not planned to have PSV operations during the night. By "night", it is meant the period from 7 p.m. to 7 a.m. Thus, time windows have fixed duration of 12 h: from 7 a.m. to 7 p.m. Each order has its own time window, which in turn is simply the time window of the platform that owns that order. Case a PSV violates the earliest time ET_{ih} of time window $h \in \mathcal{W}_i$ for order $i \in \mathcal{O}$, it is assumed it pays a price per hour given by ζ_i, corresponding to the charter hour rate of a large vessel. Case that vessel violates LT_{ih}, the penalty doubles. Penalty values are defined in the Appendix.

The planning horizon for any instance is 8 days, which leads to 8 time windows per order, case such an order arises from a platform that does not operate at night, otherwise, the order will have a single time window during 192 h to encompass the planning horizon. The starting time reference in the instances is zero, so the time windows appear as a sequence $[7, 19], [31, 43], \ldots, [175, 187]$, case no operation is allowed at night, or as single time window, $[0, 192]$, otherwise. The penalty costs for violating a time window are also given as instance data. Two PSV sizes are used in the instances: medium (`M`) and large (`L`). Vessel navigation times are calculated using geodesic distances among the platforms and historical velocities developed in real operations.

The naming for each instance involves the number of orders, platforms, PSVs, and a suffix. As an example, `5n-3c-1k-11r_4s_BCi` corresponds to an instance with 5 orders (n), 3 platforms (`c`), 1 PSV (`k`), for cluster `BCi`. The identifiers `11r` and `4s` are just for instance design control. As another example, the instance `8n-6c-2k-11r_4s_BCbBCc` has 8 orders, 6 platforms, 2 PSVs, and considers clusters `BCb` and `BCc` as if they were a single one. The instances designed and the size of PSVs they include are summarized in Table 6.2, in which #ord, #pla, and #psv stand for number of orders, platforms, and PSVs, respectively.

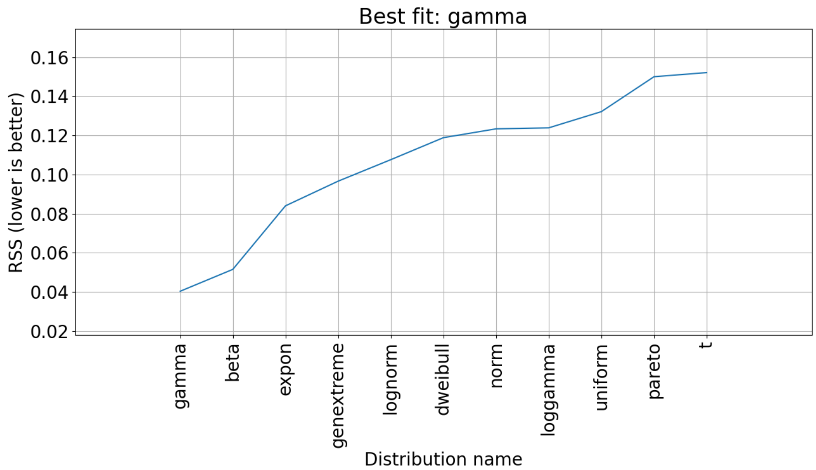
Tabela 6.2: Instances.

Name	#ord	#pla	#psv	Fleet profile	Cluster
5n-3c-1k-11r_4s_BCi	5	3	1	L	BCi
6n-4c-1k-11r_4s_BCd	6	4	1	L	BCd
8n-6c-2k-11r_4s_BCbBCc	8	6	2	L, M	BCb, BCc
10n-7c-2k-11r_4s_BCdBCi	10	7	2	L, M	BCd, BCi

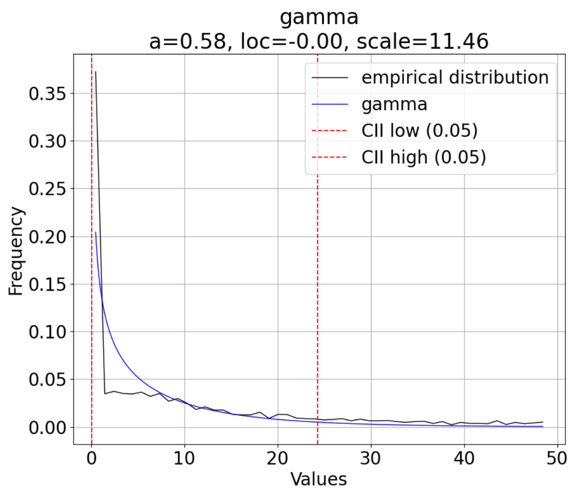
The two instances with a single cluster each are those by default defined by the operator. They were included to report the performance of the model on some of the default clusters. The other two instances were created in a manner to integrate clusters that are close to each other, as well as to allow the investigation of the model's performance over larger instances than those default.

Besides, as there are usually three vessel departures per day, the instances with two clusters serve the purpose of assessing the possibility of savings from routing clusters together, instead of separately, default case that inevitably uses more PSVs (there will be more departures). Such savings can arise either from alternative route configurations, or even from the elimination of a vessel departure, case a single PSV is able to fully attend two clusters, for example.

Regarding the random scenarios, they are designed as follows. The stochastic parameter for offshore delays in hours due to adverse environmental conditions fit to a gamma-distribution with shape $a = 0.58$ and scale $a = 11.46$ parameters, denoted by $S_{i\omega} \sim \Gamma(a, b) \equiv \text{Gamma}(0.58, 11.46)$, $i \in \mathcal{O}$, $\omega \in \Omega$. Such a distribution resulted as the best fit from historical data. Figure 6.2 presents the results of the data fitting.



(a) Best fit from smallest RSS (residual sum of squares).



(b) Gamma distribution shape.

Figura 6.2: Distribution fit results for the stochastic parameter of the problem.

A scenario refers to sampling a delay value for each of the orders of an instance. As an example, consider the instance `5n-3c-1k-11r_4s_BCi`, for which Table 6.3 shows $M = 2$ independent scenario samples of $|\Omega| = 5$ scenarios $\omega \in \{1, 2, 3, 4, 5\}$ each, generated for the five orders using different seeds per scenario group.

There were generated $n_{\text{bins}} \times M \times n_{\text{sg}}$ instances in total, in which $n_{\text{bins}} = 4$ is the number of basic instances, $M = 10$ is the number of scenario samples, and $n_{\text{sg}} = 5$ is the number of scenario samples' groups, totaling 200 instances. Another understanding for this number is that each of the 4 basic instances was exposed to

Tabela 6.3: Example of scenario samples generated for instance `5n-3c-1k-11r_-4s_BCi`. Delays sampled from $\Gamma(0.58, 11.46)$.

(a) Sample 1.

ω	Delay per order (h).				
	1	2	3	4	5
1	0	0	1.94	0	0
2	0	0	1.25	5.79	0
3	0	1.11	6.78	0	0
4	0	0	0	0	0
5	5.71	0	1.43	1.23	0

(b) Sample 2.

ω	Delay per order (h).				
	1	2	3	4	5
1	0	0	0.81	0	0
2	0	14.75	2.88	0	0
3	0	9.01	0	0	0
4	0	7.02	0	0	0
5	0	0	0	0	26.53

50 different uncertainty scenarios in this computational study.

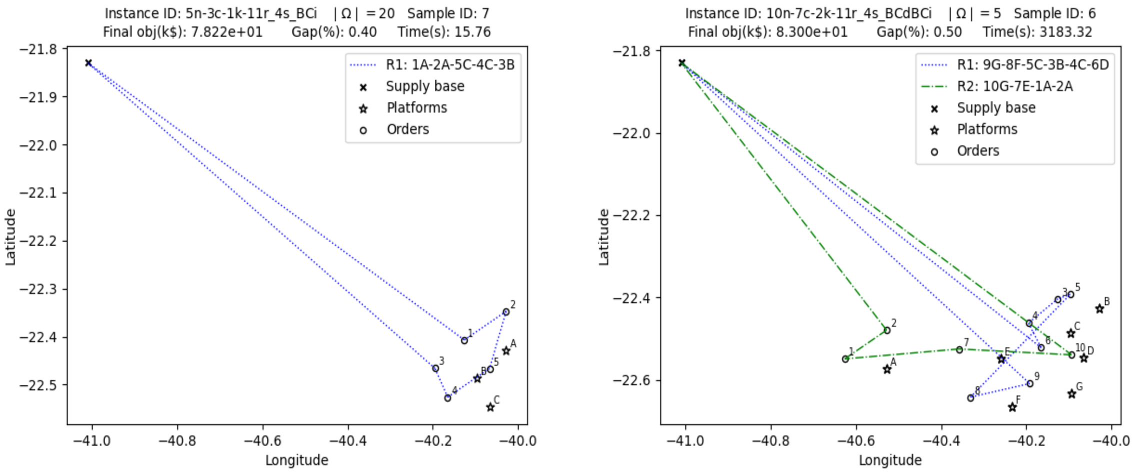
The number of scenario samples' groups used in the SAA parameters was $N \in \{5, 10, 15, 20, 25\}$, whereas the large sample value was $N' = 2000$. The value of the large scenario sample was determined empirically from two series of tests: $N' \in \{100, 200, \ldots, 900\}$, then $N' \in \{1000, 2000, \ldots, 10000\}$. It was observed that the upper bound stabilized satisfactorily from $N' = 2000$ scenarios, being meaningless to employ even larger values in the complete computational study.

6.2.2 Performance

This section demonstrates the overall performance of the approach developed to solve the s-PSVRSP. However, it is first convenient to illustrate a few representative solutions. The Figure 6.3 displays two examples of optimal routing solutions obtained with the approach developed. Those examples plot the location of the platforms and fictitious locations of the orders associated with them. This was done to turn the sequence of orders visited more easily visible in the plot.

For example, in Figure 6.3a, the platform A requested two orders numbered as 1 and 2, which share coordinates with A. The route in this figure appears in the legend as "R1-- which stands for "Route 1-- together with an association between platform and order attended. Figure 6.3b displays a final solution with two routes. This solution has two interesting aspects. First, platform C was fully attended with two visits. Second, platform G had its orders attended in different routes. Such a solution reveals that routing orders, instead of platforms, offers more flexibility to the offshore services.

Table 6.4 organizes results for instances pivoted by number of orders #ord, which in turn already indicates what basic instance is in analysis. The instances resulted in MILP models ranging from 172 non-negative real variables, 77 binary variables, and 628 constraints – case of instances with 5 orders and 5 scenarios – to 2779 non-negative real variables, 354 binary variables, and 14477 constraints – case of



(a) 5-order size instance, one PSV (`L`).
(b) 10-order size instance, two PSVs (`L`, `M`).

Figura 6.3: Routing examples.

instances with 10 orders and 25 scenarios. In this table, #ins, #opt, and #tlf stand for number of instances, number of instances deemed optimal, and number of instances time limit feasible (run ended with some incumbent found), respectively.

Tabela 6.4: Results grouped by number of orders.

#ord	#ins	#opt	time (s)	#tlf	gap (%)
5	50	50	16.5	–	–
6	50	50	26.3	–	–
8	50	50	1111.3	–	–
10	50	5	2304.0	45	8.7
Total	200	155	446.6	45	8.7

This table indicates that the model solved to optimality 77.5% of the instances in roughly 450 seconds, on average. Only 10% of the instances with 10 orders were solved to optimality, revealing that 45 instances could not be solved within the time limit of 1 h made available. The time needed to solve instances is smaller than 30 seconds up to 6 orders, whereas is grows drastically by two orders of magnitude for 8 and 10-order instance sizes. Table 6.5 presents additional results devised according to the fleet in use.

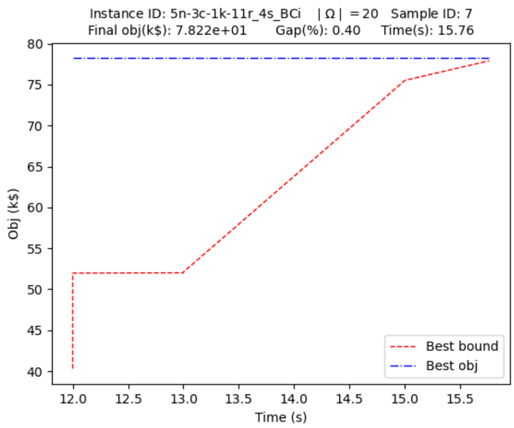
Tabela 6.5: Results grouped by fleet profile.

#psv	Fleet	#ins	#opt	time (s)	#tlf	gap (%)
1	`L`	100	100	21.4	–	–
2	`LM`	100	55	1219.7	45	8.7
Total		200	155	446.6	45	8.7

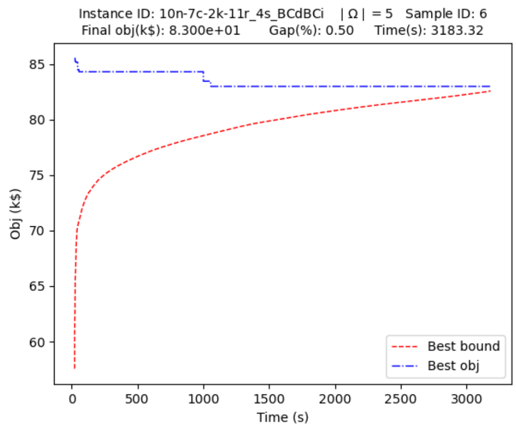
This table shows that 100% of the instances with a single large PSV were solved to optimality on average in 21.4 seconds. This means that usual clusters designed by the operator with one dedicated PSV per cluster can be solved quickly from a model that includes uncertainty in its formulation.

However, joining two clusters in a single one and offering two PSVs to them turned out to be a harder problem, as only 55% of the instances were solved optimally. In addition to that, the average time spent to solve them was two orders of magnitude greater than that for a single PSV. Even though, it is still less than 30 minutes, half of the time limit imposed.

Much of this time increase arises from some model's loss of tractability due to a graph with more nodes (orders) to be handled, the existence of two PSVs, which increase the number of binary variables and, also, because of the insertion of the constraints 6.16 – 6.17 and associated binary variables in the model, which are responsible for avoiding two PSVs operating simultaneously at a platform. Figure 6.4 presents how the problem bounds evolve over time in the solution process for instances previously presented in Figure 6.3. It clearly indicates the greater complexity in larger instances regarding the slow increase in the lower bound, case of Figure 6.4b.



(a) Bounds for 5-order size instance.



(b) Bounds for 10-order size instance.

Figura 6.4: Evolution of bounds over time.

6.2.3 Effect of the SAA method

This section presents the results achieved from the application of the SAA method to the s-PSVRSP. Table 6.6 shows the results pivoted by number of orders #ord and and by scenario sample sizes ranging according to $N \in \{5, 10, 15, 20, 25\}$.

The solution time is nearly constant for 5-order instances, regardless of N. Some time fluctuation appears for instances with 6 orders, but still small, less than 30 seconds, on average. The influence of the number of scenarios is clear for 8-order

Tabela 6.6: Results pivoted by number of orders and scenarios.

N ($M = 10$)	#ins	#opt	time (s)	#tlf	gap (%)
5	10	10	16.8	–	–
10	10	10	16.6	–	–
15	10	10	16.4	–	–
20	10	10	16.4	–	–
25	10	10	16.3	–	–
#ord = 5	50	50	16.5	–	–
5	10	10	23.8	–	–
10	10	10	26.9	–	–
15	10	10	24.5	–	–
20	10	10	28.2	–	–
25	10	10	27.8	–	–
#ord = 6	50	50	26.3	–	–
5	10	10	869.5	–	–
10	10	10	610.3	–	–
15	10	10	862.9	–	–
20	10	10	1331.0	–	–
25	10	10	1882.8	–	–
#ord = 8	50	50	1111.3	–	–
5	10	5	2304.0	5	2.7
10	10	–	–	10	5.2
15	10	–	–	10	9.1
20	10	–	–	10	10.7
25	10	–	–	10	12.6
#ord = 10	50	5	2304.0	45	8.7
Total	200	155	446.6	45	8.7

size instances. The solution time starts around 900 seconds for 5 scenarios and more than doubles at 25 scenarios.

For 10-order size instances, optimal solutions appear only for 5 instances and 5 scenarios, taking on average 2304 seconds to be solved. The 45 instances with best known solutions within 1 h of time limit behaved as expected: greater N values, despite better approximate the real life, damage the problem's tractability, consequently producing greater final gaps.

Table 6.7 shows results pivoted by number of PSVs and scenarios. All instances with a single PSV were solved to optimality in a nearly constant time. Instances with 2 PSVs presented more variability in the solution time, ranging from 610.2 seconds for 10 scenarios to 1882.8 seconds for 25 scenarios, revealing the complexity added by increasing the number of vessels in the problem.

Tabela 6.7: Results pivoted by number of PSVs and scenarios.

N ($M = 10$)	#ins	#opt	time (s)	#tlf	gap (%)
5	20	20	20.3	–	–
10	20	20	21.7	–	–
15	20	20	20.4	–	–
20	20	20	22.4	–	–
25	20	20	22.0	–	–
#psv = 1	100	100	21.4	–	–
5	20	15	1347.7	5	2.7
10	20	10	610.3	10	5.2
15	20	10	862.9	10	9.1
20	20	10	1331.0	10	10.7
25	20	10	1882.8	10	12.6
#psv = 2	100	55	1219.7	45	8.7
Total	200	155	446.6	45	8.7

Concerning the objective function values and gap estimates produced from the application of the SSA method, the Table 6.8 presents the upper and lower bound (UB and LB) estimates $\hat{z}_{N'}(\hat{x})$ and $\bar{z}_N$, respectively, their standard deviations $\hat{\sigma}_{\hat{z}_{N'}(\hat{x})}$ and $\hat{\sigma}_{\bar{z}_N}$, the absolute optimality $\text{gap}_{\text{abs}} = \hat{z}_{N'}(\hat{x}) - \bar{z}_N$ obtained from those bounds, its standard deviation $\hat{\sigma}_{\text{gap}_{\text{abs}}} = \hat{\sigma}_{\hat{z}_{N'}(\hat{x}) - \bar{z}_N}$, and the associated computing times. The estimates are based on $N' = 2000$. In addition to that, it is also provided the relative optimality gap defined by:

$$\text{gap}_{\text{rel}}(\%) = \frac{UB - LB}{UB} \times 100 = \frac{\hat{z}_{N'}(\hat{x}) - \bar{z}_N}{\hat{z}_{N'}(\hat{x})} \times 100, \tag{6.38}$$

Table 6.8 serves the purpose of demonstrating how the upper and lower bound estimates evolve as N increases. One can note that for greater N values, $\bar{z}_N$ tends to increase, whereas, given a sufficiently large scenario sample size, i.e., $N' = 2000$, the values of $\hat{z}_{N'}(\hat{x})$ either stay roughly constant ($\#\text{ord} = 6$) or tend slightly to decrease ($\#\text{ord} \in \{5, 8\}$). In turn, this behavior produces decreasing gap estimates as N increases, allowing one to have estimated solution quality measurements in the form of a gap_{abs} or gap_{rel}.

Tabela 6.8: Solution quality of the SAA method with gap estimates. Only for solutions in Table 6.4 that are optimal. Currency symbol \$ stands for USD.

N ($M = 10$)	$\hat{z}_{N'}(\hat{x})$ (\$ × 10^3)	$\hat{\sigma}_{\hat{z}_{N'}(\hat{x})}$ (\$ × 10^3)	$\bar{z}_N$ (\$ × 10^3)	$\hat{\sigma}_{\bar{z}_N}$ (\$ × 10^3)	gap_{abs} (\$ × 10^3)	$\hat{\sigma}_{\text{gap}_{\text{abs}}}$ (\$ × 10^3)	gap_{rel} (%)	$\text{time}_{\text{total}}$ (s)	$\text{time}_{\bar{z}_N}$ (s)
5	86.036	0.786	70.652	2.334	15.385	2.463	17.9	1501.8	1389.2
10	84.512	1.067	75.527	1.635	8.985	1.953	10.6	2505.3	2390.2
15	84.526	0.767	79.744	2.167	4.782	2.299	5.7	3544.8	3433.7
20	83.279	0.639	77.671	2.326	5.608	2.412	6.7	4534.8	4423.0
25	83.598	0.736	79.555	2.006	4.043	2.137	4.8	5499.5	5385.3
$\#\text{ord} = 5, \#\text{psv} = 1$									
5	82.224	0.670	67.864	1.403	14.360	1.555	17.5	1751.2	1605.7
10	84.230	0.927	74.388	2.593	9.842	2.754	11.7	2926.3	2778.4
15	82.581	0.614	78.819	3.485	3.762	3.539	4.6	4024.6	3881.4
20	82.982	0.803	76.927	1.819	6.056	1.988	7.3	5247.1	5099.8
25	83.153	0.786	81.192	2.493	1.961	2.614	2.4	6110.3	5968.8
$\#\text{ord} = 6, \#\text{psv} = 1$									
5	79.000	1.017	61.883	1.877	17.117	2.135	21.7	12132.3	11743.2
10	75.957	0.759	65.834	1.550	10.122	1.726	13.3	10158.2	9780.4
15	74.120	0.839	67.576	1.736	6.544	1.928	8.8	12047.2	11646.9
20	76.208	0.868	67.810	1.458	8.397	1.697	11.0	17609.8	17238.1
25	74.248	0.658	70.908	1.357	3.340	1.508	4.5	22824.1	22429.3
$\#\text{ord} = 8, \#\text{psv} = 2$									
5	94.376	0.837	81.159	1.670	13.217	1.868	14.0	39012.0	38473.7
$\#\text{ord} = 10, \#\text{psv} = 2$									

The SAA method is time consuming, as can be noted from the column $\text{time}_{\text{total}}$, and computationally intensive, for the requirement of solving the MILP optimization model multiples times to obtain $\bar{z}_N$, and solving the linear sub-problem $Q(x, \xi(\omega))$ thousand of times. This explains the non-negligible times even for small, low scenario size instances, such as 1501.8 seconds for $N = 5$ and $\#\text{ord} = 5$, reaching huge time values, like that for $N = 5$ and $\#\text{ord} = 10$, what corresponds to almost 11 (eleven) hours of computations (39012.0 seconds). The most time consuming part of the procedure resides in obtaining $\bar{z}_N$, which in the present experiments, responds for 94.7% of the total computing time. The complete experimental run took almost 54 hours (2 days plus 6 hours) on the hardware available.

6.2.4 Effect of introducing uncertainty

This section presents the impact of introducing uncertainty data, modeled as the stochastic parameter $S_{i\omega}, i \in \mathcal{O}, \omega \in \Omega$, in the MILP model to solve the s-PSVRSP . Table 6.9 presents the components of the VSS computation procedure, which provide means to perceive such an impact.

As a general observation, the average VSS values present a growing tendency as the size of the routing problem increases, ranging from USD 8,418.00 at 5 orders to USD 20,305.00 at 10 orders, an increase of approximately 2.5×. The extreme VSS values in the table appear for $N = 20$, $\#\text{ord} = 5$, whose economy estimated for a single-vessel routing opportunity is USD 6,369.00, and for $N = 15$, $\#\text{ord} = 8$, which corresponds to the largest savings in the table: USD 28,444.00.

Tabela 6.9: VSS results pivoted by number of orders and scenarios. Only for solutions in Table 6.4 that are optimal. Currency symbol \$ stands for USD. †Simple mean.

N ($M = 10$)	$z^{\dagger}_{\text{EV}}$ ($\$\times10^3$)	$\text{time}^{\dagger}_{z_{\text{EV}}}$ (s)	$z^{\dagger}_{\text{EEV}}$ ($\$\times10^3$)	$\text{time}^{\dagger}_{z_{\text{EEV}}}$ (s)	$z^{\dagger}_{\text{RP}}$ ($\$\times10^3$)	$\text{time}^{\dagger}_{z_{\text{RP}}}$ (s)	$VSS^{\dagger}$ ($\$\times10^3$)
5	63.758	20.2	78.727	0.1	70.652	16.8	8.075
10	66.525	20.4	82.559	0.2	75.527	16.6	7.032
15	66.409	20.5	91.521	0.3	79.744	16.4	11.777
20	65.619	20.1	84.040	0.3	77.671	16.4	6.369
25	65.987	20.6	88.390	0.4	79.555	16.3	8.835
$\#\text{ord} = 5$	65.660	20.3	85.047	0.3	76.630	16.5	8.418
5	61.812	22.9	75.861	0.1	67.864	23.8	7.998
10	64.280	22.6	85.176	0.2	74.388	26.9	10.788
15	63.966	22.6	88.855	0.3	78.819	24.5	10.036
20	63.390	22.6	88.799	0.4	76.927	28.3	11.872
25	64.304	22.0	95.389	0.5	81.193	27.8	14.196
$\#\text{ord} = 6$	63.550	22.5	86.816	0.3	75.838	26.3	10.978
5	56.006	303.7	75.848	0.3	61.883	869.5	13.964
10	56.791	366.1	84.497	0.6	65.834	610.3	18.663
15	56.943	299.5	96.020	0.9	67.576	862.9	28.444
20	56.797	389.8	87.873	1.1	67.810	1331.0	20.063
25	57.561	356.8	91.297	1.3	70.908	1882.8	20.389
$\#\text{ord} = 8$	56.820	343.2	87.107	0.9	66.802	1111.3	20.305
5	74.413	747.0	87.444	0.7	79.601	2304.0	7.844
$\#\text{ord} = 10$	74.413	747.0	87.444	0.7	79.601	2304.0	7.844

Another important observation is that $z_{\text{EV}} \leqslant z_{\text{RP}}$, which is expected, but it could inadvertently lead to regard the recourse problem solution as a more costly option, hence, undesired. Or, that said in another way, using the expected value problem's solution is cheaper, therefore better. That's a unsuitable interpretation, since the value of introducing uncertainty in the modeling is precisely to achieve the best average cost performance for upcoming scenarios.

Computing the VSS is time consuming, because the time to solve the expected value problem is not negligible, ranging from 20.2 seconds to approximately 12.5 minutes (747.0 seconds). In fact, it can be very difficult to solve too. The total computing time in the entire experimentation with VSS was 32.1 hours (1 day plus roughly 8 hours) and, so far, the SAA method together with VSS computations add up to 86.1 hours of computations (3 days plus 14 hours).

An interesting effect happens with $\text{time}_{z_{\text{EV}}}$ and $\text{time}_{z_{\text{RP}}}$. For $\#\text{ord} = 5$, $\text{time}_{z_{\text{EV}}}$ is on average nearly 4 seconds greater than $\text{time}_{z_{\text{RP}}}$. At $\#\text{ord} = 6$, the opposite happens, i.e., $\text{time}_{z_{\text{RP}}}$ becomes on average nearly 4 seconds greater than $\text{time}_{z_{\text{EV}}}$. For $\#\text{ord} \in \{8, 10\}$, $\text{time}_{z_{\text{RP}}}$ turns on average one order of magnitude greater than $\text{time}_{z_{\text{EV}}}$. This shows how fast the recourse problem's complexity increases with the number of scenarios.

6.2.5 Routing costs distribution

This section reveals how the costs in the objective function of the s-PSVRSP are distributed with respect to first and second-stages. Table 6.10 presents the cost segregation per number of orders and scenario groups. The columns of that table are:

- Obj(k\$): objective function value.
- 1st(k\$): total first-stage cost.
- 2nd(k\$): total second-stage cost.
- 2nd_{wait} (k\$): second-stage cost component related to sum of all waiting times.
- $\text{2nd}_{\text{rv}}(k\$)$: second-stage cost component related to the sum of all violations of time window ready times.
- 2nd_{dv} (k\$): second-stage cost component related to the sum of all violations of time window due times.
- $\text{2nd}_{\text{delay}}$ (k\$): second-stage cost component related to the sum of all delay costs.
- $\frac{\text{2nd}}{\text{Obj}}$: ration between total second-stage costs and objective value.
- $\frac{\text{2nd}_{\text{delay}}}{\text{Obj}}$: ration between the sum of all delay costs and objective value.
- #r: number of routes in the optimal solution.

The first relevant aspect in the Table 6.10 is that the total second-stage costs represent an expressive fraction of the objective function for all N and instances, reaching on average practically 45% when #ord = 5. This percentages reveals how influential the scenario realizations are, even in the optimal solutions.

Other trait of the problem is the inevitable second-stage cost associated with delays, which manifest as realizations of the random parameter. The fraction of the delay-related costs in comparison with the objective function is significant, ranging from a minimum of 5.5%, for $N = 5$, #ord = 10, to 10.2%, for $N = 15$, #ord = 5. This indicates that there will always exist extra diesel consumption due to delays, and there is no decision making capable of eliminating this cost completely.

Tabela 6.10: Routing costs distribution in the first and second-stages. Only for solutions in Table 6.4 that are optimal. Currency symbol \$ stands for USD. †Simple mean.

N ($M = 10$)	Obj† (\$ × 10³)	1st† (\$ × 10³)	2nd† (\$ × 10³)	$2nd^{\dagger}_{wait}$ (\$ × 10³)	$2nd^{\dagger}_{rv}$ (\$ × 10³)	$2nd^{\dagger}_{dv}$ (\$ × 10³)	$2nd^{\dagger}_{delay}$ (\$ × 10³)	$\frac{2nd}{Obj}$† (%)	$\frac{2nd_{delay}}{Obj}$† (%)	#r†
5	70.652	42.622	28.030	12.015	9.771	0.755	5.489	39.2	7.6	1
10	75.527	41.839	33.688	13.280	9.282	3.432	7.694	44.4	10.1	1
15	79.744	42.029	37.715	14.132	9.920	5.417	8.245	47.0	10.2	1
20	77.671	41.485	36.186	16.720	8.664	4.053	6.749	46.2	8.6	1
25	79.555	41.717	37.838	16.710	8.175	5.750	7.202	47.3	9.0	1
n = 5	76.630	41.938	34.691	14.571	9.163	3.881	7.076	44.8	9.1	1
5	67.864	42.355	25.509	11.048	7.650	1.409	5.402	37.4	7.8	1
10	74.388	42.254	32.134	13.904	7.732	3.092	7.406	42.7	9.7	1
15	78.819	42.350	36.469	16.689	6.547	5.382	7.851	45.4	9.8	1
20	76.927	42.622	34.305	14.314	7.159	5.997	6.834	44.4	8.8	1
25	81.193	42.658	38.534	17.212	6.870	6.254	8.198	47.0	10.0	1
n = 6	75.838	42.448	33.390	14.634	7.192	4.427	7.138	43.4	9.2	1
5	61.883	46.915	14.968	5.916	3.715	0.491	4.846	23.7	7.6	2
10	65.834	47.143	18.691	9.454	2.589	1.372	5.275	28.0	7.9	2
15	67.576	47.350	20.226	8.178	3.721	2.662	5.666	29.5	8.3	2
20	67.810	47.064	20.746	7.984	4.908	2.813	5.041	30.3	7.4	2
25	70.908	47.254	23.654	9.001	4.003	4.856	5.795	33.1	8.1	2
n = 8	66.802	47.145	19.657	8.107	3.787	2.439	5.325	28.9	7.9	2
5	79.601	65.035	14.566	6.196	3.568	0.438	4.364	18.2	5.5	2
n = 10	79.601	65.035	14.566	6.196	3.568	0.438	4.364	18.2	5.5	2

At last, the time window violation costs indicate that violating the earliest time window moment is a recourse action more expensive, in the majority of the cases, than violating the latest time. There is no surprise in it, as the penalty for violating the latest time is twice that of an earliest moment, what makes the solution process to avoid solutions in which the offshore services finish after the window closing moments. This in turn provokes more waiting times before starting a service, leading to relevant second-stage costs related to waiting.

6.3 Discussion on the practical application

The approach developed in this dissertation to solve the s-PSVRSP is relevant to practical use for some reasons. First, it provides a formal decision method to design optimal routes in a small time for operator's default cluster sizes of 3 and 4 platforms – i.e., 5 to 6 orders, usually – in opposite to traditional spreadsheets employed in routes' planning. For 6-platform size groups and 2 vessels, still not practiced by the operator, the solution time increases considerably, but it continues acceptable for operational purposes. Moreover, these routes present economical gains for taking into account exogenous uncertainty, what is demonstrated from the VSS indicator. At 7 platforms per routing and 2 vessels, the experimental results indicate that the approach developed is not recommended for operational use.

Secondly, despite the option for modeling the decisions associated to orders' arrival and departures time as second-stage recourse actions, which in turn prohibits one to define in the first-stage when the service will take place in time, the approach adopted for the s-PSVRSP still preserves some scheduling predictability, inasmuch as time window related decisions are made at the moment that the route is defined. Hence, operational personnel can be notified in advance about what will be the time windows selected for each order's service.

At last, the model developed can produce routes that violate time windows, at the price of penalization costs, whenever this is the best decision to make. This results operational flexibility to the logistics system in relation to anticipating or postponing some orders' service, at a price. An aspect of the model that helps in mitigating the violation cost associated with the earliest time window moment is the possibility to wait before starting a service, since the fuel cost per hour is expected to be much smaller than a violation penalty. Be noted that defining penalty costs may not be a simple task in real life, since it requires an agreement on how much will be charged for having a time window violated.

Capítulo 7

Conclusions

These chapter presents a consolidated view of the research topics and results achieved in this dissertation.

7.1 Regarding the current work

This dissertation introduced the *platform supply vessel operations planning problem* (PSVOPP), which appears in offshore logistics for oil and gas explorations and production activities. This problem was separated in two branches. One of them related to the tactical task of optimally organizing maritime platforms into smaller groups, called *clusters*, so that the onshore services at the supply base and the supply network backwards, responsible for delivering and collecting cargoes at the base, can be plan in advance the transport of cargoes associated with a specific cluster. This branch was named *maritime platforms clustering problem* (MPCP). In order to solve instances of the MPCP, it was proposed an MILP model that resembles an *m-dimensional multiple knapsack problem*, in which platforms are m-dimensional items and a set of PSVs plays the role of multiple-compartment multiple knapsacks.

A set of 60 realistic instances was artificially designed and used to verify the performance of the MILP model using a commercial solver, resulting in 37% of the instances solved to optimality within 250 seconds, on average, whereas the remainder 63% of them presented good quality solutions with average gap 2.7%, within 3600 seconds set as run-time limit. The MILP model also entails a composite objective function that minimizes two conflicting objectives weighed by $\alpha \in \{0, 0.25, 0.5, 0.75, 1\}$. First, the number of cluster, and second, the supply base berth time that each of these clusters impose at the base for loading of cargoes. Among the instances with $\alpha = 0.75$, the solution approach succeeded optimally in 67% of them within roughly 500 seconds on average, providing a suitable balance with respect to minimizing the number of clusters, without disregarding the associated berth times that these clusters produce. Among the remainder 33%, the average gap presented was 0.7%, which means good quality solutions as well.

The second branch covered operational decisions related to routing and scheduling of platform supply vessels (PSVs). To cope with this matter, two MILP models for routing were developed. The first one includes several features from the real life problem of PSVs' routing, including heterogeneous and multiple-compartment fleet, multiple commodity types – some of them competing for the same compartment space – and scheduling aspects such as hard time windows and the possibility to determine routes with multiple visits per platform and plan in advance for multiple trips. This problem was named *deterministic platform supply vessel routing and scheduling problem* (d-PSVRSP).

A set of 3600 realistic instances was artificially designed and used to verify the performance of the MILP model using a commercial solver. Moreover, the solution process of this model also employed cutting planes in the form of *rounded capacity inequalities* (RCIs), whose development was not a dissertation's contribution, just their conceptualization as cuts and application in the MILP model. Experiments revealed that using RCIs increased the number of optimal solutions in approximately 30%, i.e., from 1989 (55%) to 2563 instances (71%) solved optimally out of 3600. The sub-optimal solutions presented average gap of 10.1%. The MILP model also considers a composite objective function α-weighed, which minimizes two conflicting objectives: consumption of fuel by the vessels in use and route duration. Once more, adopting $\alpha = 0.75$ showed as interesting choice to cope with both objectives.

The second routing model, despite embracing a smaller number of real life features, still provides good coverage of the real operational scenario and, in addition to that, includes uncertainty data in its MILP model formulated as a deterministic-equivalent program from a two-stage stochastic program with recourse. This problem was called *stochastic* PSVRSP (s-PSVRSP). The uncertainty data consists of operational delays – originated from adverse environmental conditions that temporarily interrupt offshore services – unknown at the moment that the first-stage decisions related to routing and selection of soft time windows need to be made, but that unfold in the second-stage when the route is ongoing, what in turn allows that recourse decisions, modeled as time window violations and waiting times before starting an offshore service, be made.

Solving the s-PSVRSP for a large number of scenarios is intractable. Hence, the *sample average approximation method* (SAA) was employed, accompanied with a procedure to statistically estimate the quality of the solutions achieved from estimates for absolute and relative optimality gaps, since the SAA method provides only an approximation for the solution of the so-called “true"stochastic optimization problem from samples with a countable number of scenarios. A set of 200 realistic instances was artificially designed and used to verify the performance of the MILP model using a commercial solver. By using the SAA method, it was possible to solve optimally 77.5% of the instances in roughly 450 seconds, on average, whereas the remainder 22.5% achieved the average gap of 8.7% by the end of the run-time limit set.

Regarding the quality estimation for the optimal objective function values yielded by the SAA method with respect to their true counterpart, i.e., the original stochastic program, it was possible to achieve gap values inferior to 5% in 75% of the instances (150), and inferior to 14% in a overall view. As a mechanism to measure the benefit from introducing uncertainty in the model, it was computed the *value of the stochastic solution* (VSS), which measures how much is saved for considering random data in the model. Experiments revealed savings in USD ranging from 6,369.00 to 28,444.00 per route, which demonstrates the economical relevance of turning uncertainty data part of the model.

At last, the results achieved with all three MILP models in the approaches developed demonstrate that they are suitable for real use in offshore oil and gas logistics operations related to transportation of cargoes, as such approaches provided good quality solutions for several instances whose sizes are similar to those seen in practice, concerning the number of platforms, orders, and PSVs.

7.2 Suggestions for future work

This this section points out a few directions for future works that can build upon the contributions of this dissertation. Such suggestions are itemized as follows.

1. On the MPCP

   - Consider other clustering metrics. For instance, drilling rigs construct wells in phases, which can lead to different cargo demand profiles per phase. It could be relevant to form cluster according to phase-related cargo demand profiles.
   - Extend the model to introduce more detailed berth scheduling decisions with associated constraints, such as limited number of berths, aiming to provide more fine-grained plans for supply base operations.
   - Introduce uncertainty in the model, for instance to cope with demand fluctuations.
   - Consider different MILP formulations and combinations of heuristic and exact methods.
   - Introduce more specific cluster formation costs.

2. On the routing problems d-PSVRSP and s-PSVRSP

    - Develop heuristic and/or other exact methods that allow one to solve larger instances, case in which the models performed poorly.
    - Introduce decisions regarding PSVs' speed.

3. On integration

    - Design an integrated solution method involving the clustering and routing models on a rolling-horizon based form, possibly in an environment where uncertainty can be dynamically evaluated, such as stochastic simulation models.

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Appendix

This section formally defines additional data pertinent to the MILP models previously introduced.

Vessel's capacity factor. Constraints (5.36) involve a capacity-related factor per vessel defined as:

$$\xi^k = \sum_{q \in \mathcal{Q}} RQ_q^k \, WQ_q \qquad \forall k \in \mathcal{V} \tag{7.1}$$

The parameters and sets necessary to devise (7.1) are defined as follows:

M_{qc}	Maximum demand of the platform $c \in \mathcal{C}$ with respect to the compartment $q \in \mathcal{Q}$. Defined as: $M_{qc} = \max\left\{\sum_{i \in \mathcal{O}:c^i=c} \sum_{p^i \in \mathcal{P}_q \cap \mathcal{P}_-} \lvert D_i \rvert, \sum_{i \in \mathcal{O}:c^i=c} \sum_{p^i \in \mathcal{P}_q \cap \mathcal{P}_+} D_i\right\}$.
$\overline{Q}_q$	Average capacity with respect to $q \in \mathcal{Q}$. Defined as: $\overline{Q}_q = \sum_{k \in \mathcal{V}} Q_q^k / \lvert \mathcal{V} \rvert$.
RQ_q^k	Relative capacity of $q \in \mathcal{Q}$ for $k \in \mathcal{V}$. Defined as: $RQ_q^k = Q_q^k / \overline{Q}_q$.
RDM_{qc}	Relative demand of $c \in \mathcal{C}$ regarding $q \in \mathcal{Q}$. Defined as: $RDM_{qc} = M_{qc} / \overline{Q}_q$.
CDM_q	Consolidated relative demand of $q \in \mathcal{Q}$. Defined as: $CDM_q = \sum_{c \in \mathcal{C}} RDM_{qc}$.
WQ_q	Relative weight of compartment $q \in \mathcal{Q}$. Defined as: $WQ_q = CDM_q / \sum_{q \in \mathcal{Q}} CDM_q$.

Time-to-cost conversion factor. The parameter η monetizes f_2 in (5.37) so that the resulting value ηf_2 and f_1 be commensurable. The definition of η is:

$$\eta = \begin{cases} -, & \text{if } \alpha = 1 \\ 1, & \text{if } \alpha = 0 \\ \frac{UB|_{\alpha=1}}{UB|_{\alpha=0}}, & \text{if } \alpha \in \,]0,1[\end{cases} \tag{7.2}$$

In equation (7.2), η does not need to be defined when $\alpha = 1$, since in this case only the cumulative fuel cost f_1 is the minimizing target. Then it is assumed $\eta = 1$ if the vessels' cumulative utilization time, i.e., f_2, is solely what should be minimized. $UB|_{\alpha=1}$ and $UB|_{\alpha=0}$ stand for the best upper bound obtained when running a problem's instance twice: one time for $\alpha = 1$ and another fixing $\alpha = 0$. Hence, the relation $\frac{UB|_{\alpha=1}}{UB|_{\alpha=0}}$ yields a cost per time factor that properly allows to monetize f_2.

Big-Ms. Definition of sufficiently large numbers.

M_1 Big-M value used in constraints 4.11 and 4.13. Defined as :
$M_1 = \max\left\{\sum_{i\in\mathcal{C}}\sum_{p\in\mathcal{P}_1}\sigma_p L_{ip}, \sum_{i\in\mathcal{C}}\sum_{p\in\mathcal{P}_2}\sigma_p L_{ip}\right\}$.

M_{ij} Big-M used in constraints (5.11). Defined as:

$$M_{ij} = \begin{cases} \max_{w\in\mathcal{W}_j}\{LT_{jw}\} - ST_j, \text{ if } j \in \mathcal{O}\setminus\mathcal{O}_- \\ DT_j - \sum_{p\in\mathcal{P}_-}\mathbb{1}_{p=p^j}|D_j|\sigma_p - \min_{\substack{k\in\mathcal{V}\\ i'\in\mathcal{O}}}\{T^k_{i'h^{k+1}}\} - ST_j, \text{ if } j\in\mathcal{O}_- \\ \max\{\overline{AT}, \max_{i'\in\mathcal{O}}\{\overline{DT}_{i'} + \max_{k\in\mathcal{V}}\{T^k_{i'j}\}\}\}, \text{ if } j\in\bigcup_{k\in\mathcal{V}}\underline{\mathcal{N}}^k_0 \end{cases},$$

in which $\overline{DT}_{i'} = \begin{cases} \max_{w\in\mathcal{W}_{i'}}\{LT_{i'w}\}, \text{ if } i'\in\mathcal{O}\setminus\mathcal{O}_- \\ DT_{i'} - \sum_{p\in\mathcal{P}_-}\mathbb{1}_{p=p^{i'}}|D_{i'}|\sigma_p - \min_{\substack{k\in\mathcal{V}\\ j'\in\mathcal{O}}}\left\{T^k_{j'h^{k+1}}\right\}, \text{ if } i'\in\mathcal{O}_- \end{cases}$
and $\overline{AT} = \max_{k\in\mathcal{V}}\{AT^k\}$.

$M_{2,i}$ Big-M used in constraints (5.26). Defined as:
$M_{2,i} = DT_i - \sum_{p\in\mathcal{P}_-}\mathbb{1}_{p=p^i}|D_i|\sigma_p - \min_{\substack{k\in\mathcal{V}\\ j\in\mathcal{O}}}\left\{T^k_{jh^{k+1}}\right\} - ST_i,\ \forall i\in\mathcal{O}_-$.

M^k_3 Big-M used in constraints (5.27) and (5.29). Defined as:
$M^k_3 = \max\left\{\max_{i\in\mathcal{O}\setminus\mathcal{O}_-}\{\max_{w\in\mathcal{W}_i}\{LT_{iw}\} - ST_i\}, \max_{i\in\mathcal{O}_-}\left\{\overline{DT}^k_i - ST_i\right\}\right\} - \min_{j\in\mathcal{O}}\left\{T^k_{h^k+1,j}\right\}$, $\forall k\in\mathcal{V}$, in which $\overline{DT}^k_i = DT_i - \sum_{p\in\mathcal{P}_-}\mathbb{1}_{p=p^i}|D_i|\sigma_p - \min_{j\in\mathcal{O}}\left\{T^k_{jh^{k+1}}\right\}$.

$M_{4,i}$ Big-M used in constraints (5.28). Defined as:
$M_{4,i} = DT_i - \sum_{p\in\mathcal{P}_-}\mathbb{1}_{p=p^i}|D_i|\sigma_p - \min_{\substack{k\in\mathcal{V}\\ j\in\mathcal{O}}}\left\{T^k_{jh^{k+1}}\right\},\ \forall i\in\mathcal{O}_-$.

M^k_5 Big-M used in constraints (5.30). Defined as:
$M^k_5 = \max\left\{\max_{\substack{i\in\mathcal{O}\setminus\mathcal{O}_-\\ w\in\mathcal{W}_i}}\{LT_{iw}\} + \max_{i\in\mathcal{O}}\{T^k_{ih^{k+1}}\}, \max_{i\in\mathcal{O}_-}\{DT_i\}\right\}$, $\forall k\in\mathcal{V}$.

Vessel capacities. The net capacities for compartments of deck cargo (m^2), diesel (m^3), and water (m^3) adopted per vessel size for a PSV $k\in\mathcal{V}$ are:

$$Q^k_{\text{deck area}} = \begin{cases} 450, & \text{if } DWT^k = 1500 \\ 570, & \text{if } DWT^k = 3000 \\ 650, & \text{if } DWT^k = 4500 \end{cases} \tag{7.3}$$

$$Q^k_{\text{diesel}} = \begin{cases} 600, & \text{if } DWT^k = 1500 \\ 1000, & \text{if } DWT^k = 3000 \\ 1400, & \text{if } DWT^k = 4500 \end{cases} \tag{7.4}$$

$$Q^k_{\text{water}} = \begin{cases} 500, & \text{if } DWT^k = 1500 \\ 800, & \text{if } DWT^k = 3000 \\ 1000, & \text{if } DWT^k = 4500 \end{cases} \tag{7.5}$$

Fuel consumption costs. The diesel costs per hour for each vessel depends on its status. Diesel price per ton used was obtained from OILMONSTER (2022), and converted to cubic meters using density 852 kg/m^3, whereas vessels' consumption in m^3/h are typical values for a large PSV 4500. Consumption values for vessel sizes S and M were assumed proportional in DWT from a large PSV 4500. The hourly costs in USD/h set in the instances for a PSV $k \in \mathcal{V}$ are:

$$\theta^k = \varphi^k = \begin{cases} 31, & \text{if } DWT^k = 1500 \\ 62, & \text{if } DWT^k = 3000 \\ 94, & \text{if } DWT^k = 4500 \end{cases} \tag{7.6}$$

$$\gamma^k = \begin{cases} 242, & \text{if } DWT^k = 1500 \\ 485, & \text{if } DWT^k = 3000 \\ 727, & \text{if } DWT^k = 4500 \end{cases} \tag{7.7}$$

$$\delta^k = \begin{cases} 184, & \text{if } DWT^k = 1500 \\ 368, & \text{if } DWT^k = 3000 \\ 551, & \text{if } DWT^k = 4500 \end{cases} \tag{7.8}$$

Setups and safe positioning. The values adopted for setups and safe positioning in hours per PSV size are:

$$SE^k = \begin{cases} 0.19, & \text{if } DWT^k = 1500 \\ 0.46, & \text{if } DWT^k = 3000 \\ 0.51, & \text{if } DWT^k = 4500 \end{cases} \tag{7.9}$$

$$SP^k = \begin{cases} 0.39, & \text{if } DWT^k = 1500 \\ 0.91, & \text{if } DWT^k = 3000 \\ 1.02, & \text{if } DWT^k = 4500 \end{cases} \tag{7.10}$$

Time window violation costs. It is assumed time window violation penalties $\zeta_i = 1,250.0$ USD/h for violating the earliest time window moment, and $\beta_i = 2\zeta_i = 2,500.0$ USD/h, for violating the latest time window moment, given $i \in \mathcal{O}$. USD stands for United States Dollars. The value for ζ corresponds to the hourly cost of a PSV chartered at a daily rate of USD $30,000.0$. This daily rate was estimated from MENDES VIANNA (2016).